Unit - 9

Indefinite And Definite Integration

Important Points

1. If $\frac{d}{dx}[F(x)+c] = f(x)$ then $\int f(x)\,dx = F(x)+c$

$\int f(x)dx$ is indefinite integral of f(x) w.r.to x where c is the arbitrary constant.

Rules of indefinite Integration

1 If f and g are integrable function on [a,b] and f+g is also integrable function on [a,b], then

$\int(f(x)+g(x))dx = \int f(x)\,dx + \int g(x)\,dx$.

If $f_1, f_2, \ldots, f_n$ an integrable function on [a,b] then

$\int(f_1(x)+f_2(x)+f_3(x)+\ldots+f_n(x))dx = \int f_1(x)dx + \int f_2(x)dx+\ldots+\int f_n(x)dx.$

2 (i) If f is integrable on [a, b] and k is the real constant then, kf is also integrable then

$\int kf(x)dx = k\int f(x)dx$

(ii) $\int [k_1 f_1(x) + k_2 f_2(x) + \ldots\ldots\ldots + k_n f_n(x)]\,dx$

$= k_1 \int f_1(x)dx + k_2 \int f_2(x)dx + \ldots\ldots\ldots + k_n \int f_n(x)\,dx$

3 If f and g are integrable functions on [a, b] then

$\int(f(x)-g(x))dx = \int f(x)\,dx - \int g(x)dx$

- Important formulae

1 $\int x^n\,dx = \frac{x^{n+1}}{n+1} + c;\ n\in R-\{-1\};\ x\in R^+$

If n = 0 then $\int dx = x + c$

2 $\int \frac{1}{x}\,dx = \log|x| + c;\ x \in R - \{0\}$

3 (i) $\int a^x\,dx = \frac{a^x}{\log_e a} + c;\ a\in R^+ - \{1\},\ x \in R$

(ii) $\int e^x dx = e^x + c;\ \forall\, x\in R$

4 $\int \sin x\,dx = -\cos x + c\ ,\ \ \forall\ x \in R$

5 $\int \cos x \, dx = \sin x + c \;, \quad \forall\; x \in R$

6 $x = \tan x + c \;, \quad x \neq (2k-1)\dfrac{\pi}{2},\; k \in Z$

7 $\int cosec^2 x \, dx = -\cot x + c \;, \quad x \neq k\pi, k \in Z$

8 $\int \sec x \tan x \, dx = \sec x + c \;, \quad x \neq (2k-1)\dfrac{\pi}{2},\; k \in Z$

9 $\int cosec\, x . \cot x d\, x = -cosec\, x + c,\; x \neq k\pi, k \in Z$

10 $\int \dfrac{1}{x^2+a^2} dx = \dfrac{1}{a}\tan^{-1}\dfrac{x}{a} + c \;, \quad a \in R - \{0\},\; x \in R$

$= -\dfrac{1}{a}\cot^{-1}\dfrac{x}{a} + c,\; a \in R - \{0\},\; x \in R$

11 $\int \dfrac{dx}{x^2 - a^2} = \dfrac{1}{2a}\log\left|\dfrac{x-a}{x+a}\right| + c,\; a \in R - \{0\},\; x \neq \pm a$

12 $\int \dfrac{dx}{a^2 - x^2} = \dfrac{1}{2a}\log\left|\dfrac{x+a}{x-a}\right| + c,\; a \in R - \{0\},\; x \neq \pm a$

13 $\int \dfrac{dx}{\sqrt{x^2 \pm k}} = \log\left| x + \sqrt{x^2 \pm k}\right| c \;, \quad |x| > |k|$

14 $\int \dfrac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\dfrac{x}{a} + c \;, \quad x \in (-a, a),\; a > 0$

$= -\cos^{-1}\dfrac{x}{a} + c, \quad x \in (-a, a);\; a > 0$

15 $\int \dfrac{1}{|x|\sqrt{x^2 - a^2}} dx = \dfrac{1}{a}\sec^{-1}\dfrac{x}{a} + c \;, \quad |x| > |a| > 0$

$= -\dfrac{1}{a} cosec^{-1}\dfrac{x}{a} + c \;, \quad |x| > |a| > 0$

16. $\int \dfrac{1}{a + bx^2} dx = \dfrac{1}{\sqrt{ab}}\tan^{-1}\left(\sqrt{\dfrac{b}{a}}\, x\right) + c,\; (a, b > 0)$

<u>Method of substitution</u>

* If $g : [\alpha, \beta] \to R$ is continuous and differentiable on (α, β)

and $g'(t)$ is continuous and non zero on (α, β) if $R_g \subset [a, b]$

and $f : [a, b] \to R$ is continuous and $x = g(t)$ then $\int f(x)\, dx = \int [f\, g(t)\, g'(t)]\, dt$

* If $\int f(x)\, dx = F(x) + c$ then $\int f(ax + b)\, dx = \frac{1}{a} F(a\alpha + b) + C,$

wherc $f : I \to R$ is continuous $(a \neq 0)$

* $\int f(x)^n f'(x) dx = \frac{[f(x)]^{n+1}}{n+1} + c, \quad (n \neq -1,\ f(x) > 0,\ f'(x) \neq 0)$

* $\int \frac{f'(x)}{f(x)} dx = \log|f(x)| + c,$ (f and f′ are continuous $f'(x) \neq 0,\ f(x) \neq 0$)

* $\int \frac{f'(x)}{\sqrt{f(x)}} dx = 2\sqrt{f(x)} + c$ (f and f′ are continuous $f'(x) \neq 0,\ f(x) \neq 0$)

17. $\int \tan x\, dx = \log|\sec x| + c$

$= -\log|\cos x| + c \qquad x \neq \frac{k\pi}{2}, k \in z$

18. $\int \cot x\, dx = \log|\sin x| + c, \quad x \neq \frac{k\pi}{2}, k \in z$

$= -\log|\operatorname{cosec} x| + c$

19. $\int \operatorname{cosec} x\, dx = \log|\operatorname{cosec} x - \cot x|\, c, \ x \neq \frac{k\pi}{2}, k \in z$

$= \log\left|\tan\frac{x}{2}\right| + c$

20. $\int \sec x\, dx = \log|\sec x + \tan x| + c, \quad x \neq \frac{k\pi}{2}, k \in z$

$= \log\left|\tan\frac{\pi}{4} + \frac{x}{2}\right| + c, \qquad x \neq \frac{k\pi}{2}, k \in z$

Integrals	Substitutions		
(i) $\sqrt{x^2 + a^2}$	$x = a\tan\theta$	or	$x = a\cot\theta$
(ii) $\sqrt{x^2 - a^2}$	$x = a\sec\theta$	or	$x = a\operatorname{cosec}\theta$
(iii) $\sqrt{a^2 - x^2}$	$x = a\sin\theta$	or	$x = a\cos\theta$

(iv) $\sqrt{\dfrac{a - x}{a + x}}$ $\qquad x = a\cos 2\theta$

(v) $\sqrt{2ax - x^2}$ $\qquad x = 2a\sin^2\theta$

(vi) $\sqrt{2ax - x^2} = \sqrt{a^2 - (x-a)^2}$ $\qquad x - a = a\sin\theta$ or $a\cos\theta$

For the integrals :

$\dfrac{1}{a + b\cos x}$, $\dfrac{1}{a + c\sin x}$ and $\dfrac{1}{a + b\cos x + c\sin x}$, taking $\tan\dfrac{x}{2} = t$

*** <u>Integration by parts</u>**

$$\int u\ v\ dx = u\int v\,dx - \int\left(\frac{du}{dx}\int v\,dx\right)dx$$

21. $\displaystyle\int\sqrt{x^2 + a^2}\ dx = \frac{x}{2}\sqrt{x^2 + a^2} - \frac{a^2}{2}\log\left|x + \sqrt{x^2 + a^2}\right| + c$

22. $\displaystyle\int\sqrt{x^2 - a^2}\ dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}\log\left|x + \sqrt{x^2 + a^2}\right| + c$

23. $\displaystyle\int\sqrt{a^2 - x^2}\ dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + c \quad (a > 0)$

24. $\displaystyle\int e^{ax}\sin bx\ dx = \frac{e^{ax}}{a^2 + b^2}\left(a\sin bx - b\cos bx\right) + c$

25. $\displaystyle\int e^{ax}\cos bx\ dx = \frac{e^{ax}}{a^2 + b^2}\left(a\cos bx + b\sin bx\right) + c$

26. $\displaystyle\int e^{ax}\sin bx\ dx = \frac{e^{ax}}{\sqrt{a^2 + b^2}}\sin\left(bx - \theta\right) + c$

$$\cos\theta = \frac{a}{\sqrt{a^2 + b^2}}\ ;\ \sin\theta = \frac{b}{a^2 + b^2}\ ;\ \theta \in (0, 2\pi)$$

27. $\displaystyle\int e^{ax}\cos bx\ dx = \frac{e^{ax}}{\sqrt{a^2 + b^2}}\cos\left(bx - \theta\right) + c$

$$\cos\theta = \frac{a}{\sqrt{a^2 + b^2}}\ ;\ \sin\theta = \frac{b}{a^2 + b^2}\ ;\ \theta \in (0, 2\pi)$$

28. $\int e^x \left(f(x) + f'(x) \, dx = e^x \; f(x) + c \right)$

Definite Integration

Limit of a Sum

1. $\int_a^b f(x)dx = \lim_{h \to 0} h \sum_{i=1}^{n} f(a + ih)$

2. $\int_a^b f(x)dx \; \lim_{n \to \infty} \frac{b-a}{n} \sum_{i=1}^{n} f\left[a + i\left(\frac{b-a}{n}\right)\right] \; Where \; h = \frac{b-a}{n}$

Fundamental theorem of definite Integration

If f is continuous on [a, b] and F is differentiable on (a, b) such that

$\forall x \in (a,b) \; if \; \frac{d}{dx}(F(x)) = f(x) \; then \int_a^b f(x) \, dx = F(b) - F(a)$

Rules of definite Integration

1 If f and g are continuous in [a, b] then $\int_a^b [f(x) + g(x)] \, dx = \int_a^b f(x)dx + \int g(x)dx$

2 If f is continuous on [a, b] and k is real constant, then $\int_a^b k \, f(n) \, dx = k \int_a^b f(x) \, dx$

3 If f is continuous on the [a, b] and a < c < b then $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$

4 $\int_a^b f(x)dx = -\int_b^a f(x)dx$

5 $\int_a^b f(x)dx = \int_a^b f(t)dt = \int_a^b f(u)du$

Theorems

1 If f is even and continuous on the [-a, a] then $\int_{-a}^{a} f(x)dx = 2\int_0^a f(x)dx$

2 If f is odd and continuous on the [-a, a] then $\int_{-a}^{a} f(x)dx = 0$

3 If f is continuous on [0, a] then $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

4 If f is continuous on [a, b] then $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

5 If f is continuous on [0, 2a] then $\int_0^{2a} f(x)dx = \int_0^a f(x)dx + \int_0^a f(2a-x)dx$

Application of Integration

1 The area A of the region bounded by the curve y=f(x), X - axis and the lines

x = a, x = b is given by A = |I|, where I = $\int_a^b f(x)dx$ or I = $\int_a^b ydx$

2 The area A of the region bounded by the curve x = g(y) and the line y = a and y = b given

by A = |I| Where I = $\int_a^b g(y)dy$ or I = $\int_a^b ydx$

3 If the curve y = f(x) intersects X - axis at (c, 0) only and a < c < b then the area of the region bounded by y = f(x), x = a, x = b and X - axis is given by

A = $|I_1|+|I_2|$ where $I_1 = \int_a^c ydx$, $I_2 = \int_c^b ydx$

4 If two curves y = $f_1(x)$ and y = $f_2(x)$ intersect each other at only two points for x = a and x = b (a $\neq$ b) then the area enclosed by them is given by

A = |I| and I = $\int_a^b (f_1(x) - f_2(x))dx$

5 If the two curves $x = g_1(y)$ and $x = g_2(y)$ intersect each other at only two points for y = a and y = b (a $\neq$ b) then the area enclosed by them is given by

A = |I| where I = $\int_a^b (g_1(y) - g_2(y))dy$

Question Bank

(Indefinite Integration)

(1) $\int \frac{dx}{1+\tan x} = \_\_\_\_\_\_\_\_ + c$

(a) $\log |\sec x + \tan x|$ (b) $2\sec^2 \frac{x}{2}$

(c) $\log |x + \sin x|$ (d) $\frac{1}{2}\left[x + \log |\sin x + \cos x|\right]$

(2) $\int \frac{e^x+1}{e^x-1} dx = \_\_\_\_\_\_\_\_ + c$

(a) $2\log \left|e^{\frac{x}{2}} - e^{\frac{-x}{2}}\right|$ (b) $2\log \left|e^{\frac{x}{2}} + e^{\frac{-x}{2}}\right|$

(c) $2\log |e^x - 1|$ (d) $\log |e^x + 1|$

(3) $\int \frac{e^{5\log x} - e^{3\log x}}{e^{4\log x} - e^{2\log x}} dx = \_\_\_\_\_\_\_\_ + c$

(a) $e \cdot 2^{-2x}$ (b) $e^3 \log_e x$ (c) $\frac{x^3}{3}$ (d) $\frac{x^2}{2}$

(4) $\int \frac{dx}{x\left(x^n+1\right)} = \_\_\_\_\_\_\_\_ + c$

(a) $\frac{1}{n}\log \left|\frac{x^n+1}{x^n}\right|$ (b) $\frac{1}{n}\log \left|\frac{x^n}{x^n+1}\right|$

(c) $\frac{1}{n}\log |x^n+1|$ (d) $\frac{1}{n}\log \left|\frac{x^n-1}{x^n}\right|$

(5) $\int \frac{\log(x+1) - \log x}{x(x+1)} dx = \_\_\_\_\_\_\_\_ + c$

(a) $\log x - \log(x+1)$ (b) $\log(x+1) - \log x$

(c) $-\frac{1}{2}\left[\log\left(\frac{x+1}{x}\right)\right]^2$ (d) $-\left[\log\left(\frac{x+1}{x}\right)\right]^2$

(6) $\int e^{\cot^{-1} x}\left(1 - \frac{x}{1+x^2}\right) dx = \_\_\_\_\_\_\_\_ + c$

(a) $\frac{1}{2} x e^{\cot^{-1} x}$ (b) $\frac{1}{2} e^{\cot^{-1} x}$ (c) $x e^{\cot^{-1} x}$ (d) $e^{\cot^{-1} x}$

(7) $\int \frac{\tan x}{\sqrt{\cos x}} dx =$ __________ $+c$

(a) $\frac{-2}{\sqrt{\cos x}}$ (b) $-\frac{1}{\sqrt{\cos x}}$ (c) $\frac{-2}{3\sqrt{\cos x}}$ (d) $\frac{-3}{2\sqrt{\cos x}}$

(8) $\int e^{4\log x}\left(x^5+1\right)^{-1} =$ __________ $+c$

(a) $\frac{1}{5}\log\left(x^4+1\right)$ (b) $-\log\left(x^4+1\right)$ (c) $\log\left(x^4+1\right)$ (d) $\frac{1}{5}\log\left(x^5+1\right)$

(9) $\int \operatorname{cosec}^3 x\ dx =$ __________ $+c$

(a) $-\frac{1}{2}\operatorname{cosec} x \cot x+\frac{1}{2}\log \left|\operatorname{cosec} x+\cot x\right|$ (b) $-\frac{1}{2}\operatorname{cosec} x \cot x$

(c) $\frac{1}{2}\operatorname{cosec} x \cot x+\frac{1}{2}\log \left|\operatorname{cosec} x+\cot x\right|$ (d) $\frac{1}{2}\operatorname{cosec} x \cot x-\frac{1}{2}\log \left|\operatorname{cosec} x+\cot x\right|$

(10) If $\int \frac{2^{1/x^2}}{x^3} dx = k2^{1/x^2}+c$ then $k =$ ________

(a) $-\frac{1}{2\log_e 2}$ (b) $-\log\ 2$ (c) -2 (d) $-\frac{1}{2}$

(11) $\int (x-1)e^{-x} dx =$ __________ $+c$

(a) xe^{x} (b) $-xe^{-x}$ (c) $-xe^{x}$ (d) xe^{-x}

(12) $\int \left(\sin(\log x)-\cos(\log x)\right) dx =$ ________ $+c$

(a) $\sin(\log x)-\cos(\log x)$ (b) $-x\ \sin(\log x)$

(c) $-x\ \cos(\log x)$ (d) $\sin(\log x)+\cos(\log x)$

(13) $\int (x+4)(x+3)^7 dx =$ __________ $+c$

(a) $\frac{(x+3)^9}{9}-\frac{(x+3)^8}{8}$ (b) $\frac{(x+3)^8(8x+33)}{72}$ (c) $\frac{(x+3)^8(8x+33)}{72}$ (d) $\frac{(x+3)^8}{8}$

(14) $\int \frac{dx}{(x+3)\sqrt{x+2}} =$ __________ $+c$

(a) $2\tan^{-1}\sqrt{x+2}$ (b) $2\tan^{-1}\sqrt{x^2+3}$ (c) $2\tan^{-1} x$ (d) $2\tan^{-1}\sqrt{x^2+2}$

(15) $\int \frac{e^x}{e^x+2+e^{-x}} =$ __________ $+c$

(a) $-\frac{1}{2}\left(e^{2x}+1\right)$ (b) $-\frac{1}{2}\left(e^{2x}+1\right)^{-1}$ (c) $-\left(e^{2x}+1\right)$ (d) $-\left(e^{2x}+1\right)^{-1}$

(16) If $\int \frac{\cos x}{\sqrt{\sin^2 x+2\sin x+1}}dx = A\log\sqrt{\sin x+1}+c$ then $A=$ ________

(a) 2 (b) 1 (c) $\frac{1}{2}$ (d) -2

(17) $\int \frac{dx}{e^x+1} =$ ________ $+c$

(a) $-\log\left|\frac{e^x+1}{e^x}\right|$ (b) $-\log\left|\frac{e^x}{e^x+1}\right|$ (c) $\log\left|\frac{e^x+1}{2e^x}\right|$ (d) $\log\left|\frac{e^{2x}}{e^x+1}\right|$

(18) $\int \frac{\cos^8 x-\sin^8 x}{1-2\sin^2 x\cos^2 x}dx =$ ________ $+c$

(a) $-\frac{\cos 2x}{2}$ (b) $-\frac{\sin 2x}{2}$ (c) $\frac{\cos 2x}{2}$ (d) $\frac{\sin 2x}{2}$

(19) $\int \frac{1}{1+(\log x)^2}d(\log x)dx =$ ________ $+c$

(a) $\frac{\tan^{-1}(\log x)}{x}$ (b) $\tan^{-1}(\log x)$ (c) $\frac{\tan^{-1}}{x}$ (d) $\tan^{-1} x$

(20) If $\int \frac{1+\cos 8x}{\cot 2x-\tan 2x}dx = A\cos 8x+C$ then A = ________

(a) $\frac{1}{16}$ (b) $-\frac{1}{8}$ (c) $-\frac{1}{16}$ (d) $\frac{1}{8}$

(21) If $\int \frac{4e^x+6e^{-x}}{9e^x-4e^{-x}}dx = Ax+B\log\left(9e^{2x}-4\right)+c$ then $A=$ ______ and $B=$ ______

(a) $\frac{3}{2},\frac{-35}{36}$ (b) $\frac{-3}{2},\frac{-35}{36}$ (c) $\frac{-3}{2},\frac{35}{36}$ (d) $\frac{3}{2},\frac{35}{36}$

(22) If $\int \frac{dx}{sm^6 x+\cos^6 x} = K\tan^{-1}\left(\frac{\tan 2x}{2}\right)+c$ then $K=$ ________

(a) $\frac{1}{2}$ (b) -1 (c) 1 (d) $-\frac{1}{2}$

(23) If $\int \frac{\sqrt{x}}{\sqrt{1-x^{3/2}}}dx = P\sqrt{1-x^{3/2}}+c$ then $P=$ ________

(a) $\frac{4}{3}$ (b) $\frac{3}{4}$ (c) $\frac{-4}{3}$ (d) $\frac{-3}{4}$

(24) $\int \frac{\sec x dx}{\sqrt{\sin(2x+\alpha)+\sin\alpha}} =$ ________ $+c$

(a) $\sqrt{2\sec\alpha(\tan x-\tan\alpha)}$ (b) $\sqrt{2\sec\alpha(\tan x+\tan\alpha)}$

(c) $\sqrt{2\sec\alpha(\cot x+\cot\alpha)}$ (d) $\sqrt{2\sec\alpha(\cot x-\cot\alpha)}$

(25) If $\int\frac{x^4+1}{x^6+1}dx=\tan^{-1}x+P\tan^{-1}x^3+c$ then $P=$ _______

(a) 3 (b) $\frac{1}{3}$ (c) $-\frac{1}{3}$ (d) -3

(26) $\int\frac{\log x-1}{(\log x)^2}dx=$ _________ $+c$

(a) $x\log x$ (b) $-x\log x$ (c) $\frac{x}{\log x}$ (d) $\frac{-x}{\log x}$

(27) $\int\frac{e^x\log\left(ex^x\right)}{x}dx=$ ________ $+c$

(a) $\frac{e^x}{x}\log x^x$ (b) $e^x\log x^x$ (c) $e^x x\log x$ (d) $\log\left(xe^x\right)$

(28) If $\int x\operatorname{cosec}^2 x\,dx=P\cdot x\cot x+Q\log|\sin x|+c$ then $P+Q=$ _______

(a) 1 (b) 2 (c) 0 (d) -1

(29) If $\int x^6\log x\,dx=Px^7\log x+Qx^7+c$ then $P+Q=$ _________

(a) $\frac{6}{49}$ (b) $-\frac{1}{49}$ (c) $\frac{1}{49}$ (d) $-\frac{6}{49}$

(30) $\int\left[\log(\log x)+\frac{1}{\log x}\right]dx=$ _________ $+c$

(a) $\frac{x}{\log(\log x)}$ (b) $x+\log(\log x)$ (c) $\log(\log x)+\frac{1}{x}$ (d) $x\log(\log x)$

(31) $\int\left(\frac{x^2+1}{x^2}\right)e^{\frac{x^2-1}{x^2}}dx=$ ___________ $+c$

(a) $e^{x-\frac{1}{x}}$ (b) $e^{x+\frac{1}{x}}$ (c) $e^{\frac{1}{x}-x}$ (d) $e^{-x-\frac{1}{x}}$

(32) $\int \frac{(x^2-1)dx}{(x^4+3x^2+1)\tan^{-1}\left(\frac{x^2+1}{x}\right)} =$ ____________ $+c$

(a) $\log\left|\tan^{-1}\left(x+\frac{1}{x}\right)\right|$ (b) $\log\left|\tan^{-1}\left(x-\frac{1}{x}\right)\right|$

(c) $\tan^{-1}\left(x+\frac{1}{x}\right)$ (d) $\tan^{-1}\left(x-\frac{1}{x}\right)$

(33) $\int \cos x\ d(\sin x) =$ ____________ $+c$

(a) $\frac{\sin 2x}{2} - x$ (b) $\frac{1}{2}\left(\frac{\sin 2x}{2} - x\right)$ (c) $\tan^{-1}\left(x+\frac{1}{x}\right)$ (d) $\tan^{-1}\left(x-\frac{1}{x}\right)$

(34) $\int \frac{e^x + xe^x}{\cos^2\left(xe^x\right)} dx =$ ____________ $+c$

(a) $\log\left|e^x + xe^x\right|$ (b) $\sec\left(xe^x\right)$ (c) $\tan\left(xe^x\right)$ (d) $\cot\left(xe^x\right)$

(35) If $\int \sin^3 x dx = A\cos^3 x + B\cos x + c$ then $A - B =$ _______

(a) $\frac{4}{3}$ (b) $\frac{-4}{3}$ (c) $\frac{1}{3}$ (d) $-\frac{1}{3}$

(36) $\int \frac{dx}{e^x + e^{-x}} =$ ___________ $+c$

(a) $\log\left|e^x + e^{-x}\right|$ (b) $\tan^{-1}\left(e^x\right)$

(c) $\log\left|e^x + 1\right|$ (d) $\tan^{-1}\left(e^{-x}\right)$

(37) $\int e^{2x + \log x} dx =$ ___________ $+c$

(a) $\frac{1}{4}(2x-1)e^{2x}$ (b) $\frac{1}{2}(2x-1)e^{2x}$

(c) $\frac{1}{4}(2x+1)e^{2x}$ (b) $\frac{1}{4}(2x+1)e^{2x}$

(38) $\int \frac{x-\sin x}{1-\cos x}dx =$ ________ $+c$

(a) $x\tan\frac{x}{2}$ (b) $-x\cot\frac{x}{2}$ (c) $\cot\frac{x}{2}$ (d) $-\cot\frac{x}{2}$

(39) $\int \frac{5+\log x}{(6+\log x)^2}dx =$ ________ $+c$

(a) $\frac{\log x}{x}$ (b) $\frac{x}{\log x+6}$ (c) $\frac{\log x+6}{x}$ (d) $x(\log x+6)$

(40) If $\int \frac{dx}{5+4\cos x} = P\tan^{-1}\left(\frac{\tan\frac{x}{2}}{3}\right)+c$ then $P:$ ______

(a) $\frac{3}{2}$ (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) $\frac{2}{3}$

(41) $\int \frac{\log x}{x^2}dx =$ ________ $+c$

(a) $\frac{-1}{x}(\log_e x+1)$ (b) $\frac{1}{x}(\log_e x+1)$ (c) $\log_e x+1$ (d) $-(1+\log_e x)$

(42) If $\int \frac{(-\sin x+\cos x)dx}{(\sin x+\cos x)\sqrt{\sin x\cos x+\sin^2 x\cos^2 x}} = -\operatorname{cosec}^{-1}[f(x)]+c$ then $f(x)=$ __

(a) $\sin 2x+1$ (b) $1-\sin 2x$ (c) $\sin 2x-1$ (d) $\cos 2x+1$

(43) If $\int \frac{\cos x dx}{\sin^3 x+\cos^3 x} = -\frac{1}{6}\log\left|\frac{z^2-z+1}{(z+1)^2}\right| - \frac{1}{\sqrt{3}}\tan^{-1}\frac{2z-1}{\sqrt{3}}+c$ then $z=$ ____

(a) $\tan x$ (b) $\cot x$ (c) $\sin x$ (d) $\cos x$

(44) $\int \sqrt{1+\sec x}dx =$ ________ $+c$

(a) $-2\sin^{-1}(2\cos x+1)$ (b) $-\sin^{-1}(2\cos x-1)$ (c) $\sin^{-1}(2\cos x-1)$ (d) $\cos^{-1}(2\cos x-1)$

(45) $\int \left(\sqrt{\tan x}+\sqrt{\cot x}\right)dx = \sqrt{2}\sin^{-1}($ _____ $)+c$

(a) $\sin x-\cos x$ (b) $\cos x-\sin x$ (c) $\sin\frac{x}{2}-\cos\frac{x}{2}$ (d) $\cos\frac{x}{2}-\sin\frac{x}{2}$

(46) $\int \frac{(x^5-x)^{1/5}dx}{x^6} =$ ________ $+c$

(a) $\frac{5}{24}\left(1-\frac{1}{x^4}\right)^{\frac{6}{5}}$ (b) $\frac{1}{24}\left(1-\frac{1}{x^4}\right)^{\frac{1}{5}}$ (c) $\frac{5}{24}\left(1-\frac{1}{x^4}\right)^{\frac{1}{5}}$ (d) $\frac{5}{24}\left(1-\frac{1}{x^4}\right)^{6}$

(47) $\int \frac{dx}{(x-1)^{\frac{3}{2}}(x-2)^{\frac{1}{2}}} =$ ________ $+c$

(a) $2\sqrt{\frac{x-1}{x-2}}$ (b) $\sqrt{\frac{x-1}{x+2}}$ (c) $2\sqrt{\frac{x-2}{x-1}}$ (d) $2\sqrt{\frac{x-1}{x+2}}$

(48) $\int \frac{x^2+1}{x^4-x^2+1}dx =$ ________ $+c$

(a) $\tan^{-1}\left(\frac{x^2+1}{x}\right)$ (b) $\tan^{-1}\left(\frac{x^2-1}{x}\right)$ (c) $\tan^{-1}(x+1)$ (d) $\tan^{-1}(x-1)$

(49) $\int \sqrt{\frac{\sin x - \sin^3 x}{1-\sin^3 x}}dx =$ ________ $+c$

(a) $\frac{2}{3}\sin^{-1}\left(\sin^{\frac{3}{2}} x\right)$ (b) $\frac{2}{3}\sin^{-1}\left(\cos^{3/2} x\right)$ (c) $\frac{-3}{2}\sin^{-1}\left(\sin^{\frac{3}{2}} x\right)$ (d) $\frac{3}{2}\sin^{-1}\left(\sin^{\frac{3}{2}} x\right)$

(50) $\int \cot^{-1}\sqrt{x}dx =$ ________ $+c$

(a) $(x+1)\cot^{-1}\sqrt{x}+\sqrt{x}$ (b) $(x+1)\cot^{-1}\sqrt{x}-\sqrt{x}$

(c) $x\cot^{-1}\sqrt{x}-\sqrt{x}$ (d) $\sqrt{x}\left(\cot^{-1}\sqrt{x}-x\right)$

(51) $\int \frac{\log x}{(1+\log x)^2}dx =$ ________ $+c$

(a) $\frac{x}{1+\log x}$ (b) $x(1+\log x)$ (c) $\frac{x}{\log x}$ (d) $x\log x + x^{-1}$

(52) $\int \frac{x^2dx}{(x^2+2)(x^3+3)} =$ ________ $+c$

(a) $\sqrt{3}\tan^{-1}\frac{x}{\sqrt{3}}+\sqrt{2}\tan^{-1}\frac{x}{\sqrt{2}}$ (b) $\sqrt{3}\tan^{-1}\frac{x}{\sqrt{3}}-\sqrt{2}\tan^{-1}\frac{x}{\sqrt{2}}$

(c) $\tan^{-1}\frac{x}{\sqrt{3}}+\sqrt{2}\tan^{-1}\frac{x}{\sqrt{2}}$ (d) $\tan^{-1}\frac{x}{\sqrt{3}}-\sqrt{2}\tan^{-1}\frac{x}{\sqrt{2}}$

(53) $\int \frac{1+x}{1+\sqrt[3]{x}}dx =$ ________ $+c$

(a) $\frac{3}{5}x^{5/3}-\frac{3}{4}x^{\frac{4}{3}}-x$ (b) $\frac{3}{5}x^{\frac{5}{3}}-\frac{3}{4}x^{\frac{4}{3}}+x$

(c) $\frac{3}{5}x^{\frac{5}{3}}+\frac{3}{4}x^{\frac{4}{3}}+x$ (d) $\frac{3}{5}x^{\frac{5}{3}}+\frac{3}{4}x^{\frac{4}{3}}-x$

(54) If $\int \frac{dx}{(1+x^2)\sqrt{1-x^2}} = \frac{1}{\sqrt{2}}\cos^{-1}[f(x)]+c$ then $f(x)=$ ________

(a) $\sqrt{\frac{1-x^2}{1+x^2}}$ (b) $\sqrt{\frac{1+x^2}{1-x^2}}$ (c) $\sqrt{\frac{x^2-1}{x^2+1}}$ (d) $\sqrt{\frac{x^2+1}{x^2-1}}$

(55) $\int \frac{\cot x dx}{\sqrt{\cos^4 x+\sin^4 x}} =$ ________ $+c$

(a) $\frac{1}{2}\log\left|\cot^2 x+\sqrt{\cot^4+1}\right|$ (b) $-\frac{1}{2}\log\left|\cot^2 x+\sqrt{\cot^4+1}\right|$

(c) $\frac{1}{2}\log\left|\tan^2 x+\sqrt{\tan^4+1}\right|$ (d) $-\frac{1}{2}\log\left|\cot x+\sqrt{\cot^4+1}\right|$

(56) $\int e^x\left(\frac{1-x}{1+x^2}\right)^2 dx =$ ___________ $+c$

(a) $e^x\left(1+x^2\right)$ (b) $\frac{e^x}{1+x^2}$ (c) $e^x\left(\frac{1-x}{1+x^2}\right)$ (d) $e^x\left(1-x^2\right)$

(57) $\int \frac{dx}{\sqrt{\cos^3 x\sin\left(x+\alpha\right)}} =$ ____________ $+c$

(a) $2\sec\alpha\sqrt{\sin\alpha+\cos\alpha\tan x}$ (b) $\sec\alpha\sqrt{\sin\alpha+\cos\alpha\tan x}$

(c) $\sqrt{\sin\alpha+\cos\alpha\tan x}$ (d) $2\sqrt{\sin\alpha+\cos\alpha\tan x}$

(58) If $\int \frac{dx}{1-\cos^4 x} = -\frac{1}{2}\cot x + A\tan^{-1}\left(f\left(x\right)\right)+c$ then $A=$ _____ and $f\left(x\right)=$ _______

(a) $-\frac{\sqrt{2}}{4}$ and $\sqrt{2}\cot x$ (b) $\sqrt{2}$ and $\sqrt{2}\tan x$

(c) $-\sqrt{2}$ and $\sqrt{2}\tan x$ (d) $\frac{1}{2\sqrt{2}}$ and $\sqrt{2}\tan x$

(59) $\int \frac{\sqrt{1-\sin x}}{1+\cos x}e^{\frac{-x}{2}}dx =$ __________ $+c$

(a) $e^{\frac{-x}{2}}\sec\frac{x}{2}$ (b) $-e^{\frac{-x}{2}}\sec\frac{x}{2}$ (c) $-2e^{\frac{-x}{2}}\sec\frac{x}{2}$ (d) $2e^{\frac{-x}{2}}\sec\frac{x}{4}$

(60) $\int \frac{dx}{\left(x+2\right)^{\frac{12}{13}}\left(x-5\right)^{\frac{14}{13}}} =$ __________ $+c$

(a) $\frac{-13}{7}\left(\frac{x+2}{x-5}\right)^{\frac{1}{13}}$ (b) $\frac{13}{7}\left(\frac{x+2}{x-5}\right)^{\frac{1}{13}}$ (c) $\frac{13}{7}\left(\frac{x-5}{x+2}\right)^{\frac{1}{13}}$ (d) $\frac{-13}{7}\left(\frac{x-5}{x-2}\right)^{\frac{1}{13}}$

(61) $\int \frac{x^2dx}{\left(x\sin x+\cos x\right)^2} =$ __________ $+c$

(a) $\frac{\sin x-x\cos x}{x\sin x+\cos x}$ (b) $\frac{\sin x+x\cos x}{x\sin x+\cos x}$ (c) $\frac{x\sin x-\cos x}{x\sin x+\cos x}$ (d) $\frac{x\sin x+\cos x}{x\sin x-\cos x}$

(62) $\int\left(1+x-\frac{1}{x}\right)e^{x+\frac{1}{x}}dx =$ __________ $+c$

(a) $\left(x+1\right)e^{x+\frac{1}{x}}$ (b) $\left(x-1\right)e^{x+\frac{1}{x}}$ (c) $-x\,e^{x+\frac{1}{x}}$ (d) $x\,e^{x+\frac{1}{x}}$

(63) If $\int \frac{5x+3}{\sqrt{x^2+4x+10}} = k_1\sqrt{x^2+4x+10} + k_2 \log\left|(x+2)+\sqrt{x^2+4x+10}\right| + c$

then $k_1 + k_2 =$ ________

(a) -1 (b) -2 (c) 1 (d) 2

(64) $\int (1-\cos x)\text{cosec}^2 x dx =$ __________ $+c$

(a) $\tan\frac{x}{2}$ (b) $\cot\frac{x}{2}$ (c) $\frac{1}{2}\tan\frac{x}{2}$ (d) $2\tan\frac{x}{2}$

(65) $\int \frac{dx}{(2\sin x + 3\cos x)^2} =$ ____________ $+c$

(a) $-\frac{1}{2\tan x+3}$ (b) $\frac{1}{2\tan x+3}$ (c) $-\frac{1}{2(2\tan x+3)}$ (d) $\frac{1}{2(2\tan x+3)}$

(66) If $f(x) = \cos x - \cos^2 x + \cos^3 x - \cos^4 x + ...$ then $\int f(x)dx =$ ______ $+c$

(a) $\tan\frac{x}{2}$ (b) $x+\tan\frac{x}{2}$ (c) $x-\frac{1}{2}\tan\frac{x}{2}$ (d) $x-\tan\frac{x}{2}$

(67) $\int \frac{e^x dx}{(e^x+2012)(e^x+2013)} =$ __________ $+c$

(a) $\log\left(\frac{e^x+2012}{e^x+2013}\right)$ (b) $\log\left(\frac{e^x+2013}{e^x+2012}\right)$ (c) $\frac{e^x+2012}{e^x+2013}$ (d) $\frac{e^x+2013}{e^x+2012}$

(68) If $\int \frac{x^{2011}\tan^{-1}(x^{2012})}{1+x^{4024}} dx = k\tan^{-1}(x^{2012}) + c$

(a) $\frac{1}{2012}$ (b) $-\frac{1}{2012}$ (c) $\frac{1}{4024}$ (d) $-\frac{1}{4024}$

(69) $\int \frac{dx}{\cos x - \sin x} =$ __________ $+c$

(a) $\frac{1}{\sqrt{2}}\log\left|\tan\left(\frac{x}{2}-\frac{3\pi}{8}\right)\right|$ (b) $\frac{1}{\sqrt{2}}\log\left|\tan\left(\frac{x}{2}+\frac{3\pi}{8}\right)\right|$

(c) $\frac{1}{\sqrt{2}}\log\left|\tan\left(\frac{x}{2}-\frac{\pi}{8}\right)\right|$ (d) $\frac{1}{\sqrt{2}}\log\left|\tan\left(\frac{x}{2}+\frac{\pi}{8}\right)\right|$

(70) If $\int \frac{\sin x\, dx}{\sin(x-\alpha)} = Ax + B\log\left|\sin(x-\alpha)\right| + c$ then $A^2 + B^2 =$ ________

(a) 1 (b) 0 (c) $\cos^2\alpha + 1$ (d) $sm^2\alpha + 1$

(71) If $\int \frac{5^x dx}{\sqrt{25^x - 1}} = k \log \left|5^x + \sqrt{25^x - 1}\right| + c$ then k = __________

(a) $\log_e^{\frac{1}{5}}$ (b) $\log_e^5$ (c) $\log_e^{25}$ (d) $\log_e^{\frac{1}{25}}$

(72) If $\int \sin^{-1}\left(\frac{2x}{1+x^2}\right)dx = f(x) - \log\left(1+x^2\right) + c$ then $f(x) =$ ________

(a) $x\tan^{-1}x$ (b) $-x\tan^{-1}x$ (c) $2x\tan^{-1}x$ (d) $-2x\tan^{-1}x$

(73) If $\int \frac{\sin^{-1}\sqrt{x} - \cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x}}dx = k \cdot \frac{1}{2}\left[\sqrt{x - x^2} - (1-2x)\sin^{-1}\sqrt{x}\right] - x + c$ then k = ___

(a) $\frac{\pi}{4}$ (b) $\frac{4}{\pi}$ (c) $\frac{\pi}{2}$ (d) $\frac{2}{\pi}$

(74) If $\int \sin\left(2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right)dx = A\sin^{-1}x + Bx\sqrt{1-x^2} + c$ then $A + B =$ ______

(a) 0 (b) $\frac{1}{2}$ (c) 1 (d) $-\frac{1}{2}$

(75) If $\int \frac{\left(1+x^n\right)^{1/n}}{x^{n+2}}dx = a\left(1+\frac{1}{x^4}\right)^b + c$ then $a + b =$ ________

(a) $\frac{6}{5}$ (b) $\frac{11}{10}$ (c) $\frac{21}{10}$ (d) $\frac{16}{13}$

(76) If $\int 5^{5^{5^x}} 5^{5^x} 5^x dx = k5^{5^{5^x}} + c$ then $k =$ ________

(a) $(\log_e 5)^{-1}$ (b) $(\log_e 5)^{-2}$ (c) $(\log_e 5)^{-3}$ (d) $(\log_e 5)^{-4}$

(77) $\int \sqrt{1+\operatorname{cosec} x}\,dx =$ __________ $+ c$

(a) $2\sin^{-1}\left(\sqrt{\cos x}\right)$ (b) $2\cos^{-1}\left(\sqrt{\sin x}\right)$ (c) $2\sin^{-1}\left(\sqrt{\sin x}\right)$ (d) $2\cos^{-1}\left(\sqrt{\cos x}\right)$

(78) $\int \frac{dx}{\sqrt{1+\operatorname{cosec}^2 x}} =$ __________ $+ c$

(a) $\sin^{-1}\left(\frac{\sin x}{\sqrt{2}}\right)$ (b) $\sin^{-1}\left(\frac{\cos x}{\sqrt{2}}\right)$ (c) $\cos^{-1}\left(\frac{\cos x}{\sqrt{2}}\right)$ (d) $\cos^{-1}\left(\frac{\sin x}{\sqrt{2}}\right)$

(79) $\int \frac{2^{\sqrt{x}}}{\sqrt{x}}dx =$ ___________ $+ c$

(a) $2^{\sqrt{x}}\log_2^e$ (b) $2^{\sqrt{x}}\log_e^2$ (c) $2^{\sqrt{x}+1}\log_2^e$ (d) $2^{\sqrt{x}+1}\log_e^2$

(80) $\int \operatorname{cosec}\left(x-\frac{\pi}{6}\right)\operatorname{cosec}\left(x-\frac{\pi}{3}\right)dx = k\left[\log\left|\sin\left(x-\frac{\pi}{6}\right)\right|-\log\left|\sin\left(x-\frac{\pi}{3}\right)\right|\right]+c$ then k = _____

(a) 2 (b) -2 (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{2}{\sqrt{3}}$

(81) $\int \frac{dx}{\left(\sin^5 x\cos^7 x\right)^{1/6}} =$ __________ $+c$

(a) $4\left(\tan x\right)^{1/4}$ (b) $6\left(\tan x\right)^{\frac{1}{6}}$ (c) $4\left(\tan x\right)^{\frac{1}{6}}$ (d) $6\left(\cot x\right)^{\frac{1}{6}}$

(82) $\int e^x\left[\frac{x^3-x-2}{\left(x^2+1\right)^2}\right]dx =$ ___________ $+c$

(a) $e^x\left(\frac{2x-1}{x^2+1}\right)$ (b) $e^x\left(\frac{x+1}{x^2+1}\right)$ (c) $e^x\left(\frac{x-1}{x^2+1}\right)$ (d) $e^x\left(\frac{2x-2}{x^2+1}\right)$

(83) $\int \frac{\left(e^x-1\right)}{\left(e^x+1\right)}\frac{dx}{\sqrt{e^x+1+e^{-x}}} =$ _________ $+c$

(a) $\tan^{-1}\left(e^x+e^{-x}\right)$ (b) $\sec^{-1}\left(e^x+e^{-x}\right)$ (c) $2\tan^{-1}\left(e^{x/2}+e^{-x/2}\right)$ (d) $2\sec^{-1}\left(e^{x/2}+e^{-x/2}\right)$

(84) $\int \frac{dx}{x^{\frac{1}{5}}\sqrt{x^{\frac{8}{5}}-1}} =$ __________ $+c$

(a) $\frac{5}{4}\log\left|x^{\frac{4}{5}}+\sqrt{x^{\frac{8}{5}}-1}\right|$ (b) $\frac{-5}{4}\log\left|x^{\frac{4}{5}}+\sqrt{x^{\frac{8}{5}}-1}\right|$

(c) $\frac{4}{5}\log\left|x^{\frac{4}{5}}+\sqrt{x^{\frac{8}{5}}-1}\right|$ (d) $\frac{-4}{5}\log\left|x^{\frac{4}{5}}+\sqrt{x^{\frac{8}{5}}-1}\right|$

(85) If $\int\left(x^{30}+x^{20}+x^{10}\right)\left(2x^{20}+3x^{10}+6\right)^{\frac{1}{10}}dx = k\left(2x^{30}+3x^{20}+6x^{10}\right)^{\frac{11}{10}}+c$ then $k =$ ___

(a) $\frac{1}{60}$ (b) $-\frac{1}{60}$ (c) $\frac{1}{66}$ (d) $-\frac{1}{66}$

(86) $\int \frac{dx}{\sqrt{(x-4)(7-x)}} =$ _________ $+c$ $(4<x<7)$

(a) $2\sin^{-1}\sqrt{\frac{x-4}{3}}$ (b) $2\cos^{-1}\sqrt{\frac{x-4}{3}}$ (c) $\frac{1}{2}\sin^{-1}\sqrt{\frac{x-4}{3}}$ (d) $-\frac{1}{2}\sin^{-1}\sqrt{\frac{x-4}{3}}$

(87) If $\int \frac{2012x+2013}{2013x+2012}dx = \frac{2012}{2013}x + k\log|2013x+2012|+c$ then $k =$ _______

(a) $\frac{4025}{2013}$ (b) $\frac{4025}{(2013)^2}$ (c) $\frac{-4025}{2013}$ (d) $\frac{-4025}{(2013)^2}$

(88) If $\int \frac{2\sin x+\cos x}{7\sin x-5\cos x}dx = ax+b\log|7\sin x-5\cos x|+c$ then $a-b=$ ________

(a) $\frac{4}{37}$ (b) $-\frac{4}{37}$ (c) $\frac{8}{37}$ (a) $-\frac{8}{37}$

(89) If $\int \frac{\cos 9x+\cos 6x}{2\cos 5x-1}dx = k_1\sin 4x + k_2\sin x + c$ then $4k_1+k_2=$ ________

(a) 1 (b) 2 (c) 4 (d) 5

(90) $\int \frac{dx}{(x\tan x+1)^2} =$ ____________ $+c$

(a) $\frac{\tan x}{x\tan x+1}$ (b) $\frac{\cot x}{x\tan x+1}$ (c) $\frac{-\tan x}{x\tan x+1}$ (d) $-\frac{1}{x\tan x+1}$

(91) $\int \sqrt{1+\sin\frac{x}{4}}dx =$ ______________ $+c$

(a) $8\left(\sin\frac{x}{8}+\cos\frac{x}{8}\right)$ (b) $\sin\frac{x}{8}+\cos\frac{x}{8}$ (c) $\frac{1}{8}\left(\sin\frac{x}{8}-\cos\frac{x}{8}\right)$ (d) $8\left(\sin\frac{x}{8}-\cos\frac{x}{8}\right)$

(92) $\int \frac{(x+1)dx}{x(1+xe^x)^2} =$ ______________ $+c$

(a) $\log\left|\frac{xe^x}{1+xe^x}\right| - \frac{1}{1+xe^x}$ (b) $\log\left|\frac{xe^x+1}{xe^x}\right| + \frac{1}{1+xe^x}$

(c) $\log\left|\frac{xe^x}{1+xe^x}\right| + \frac{1}{1+xe^x}$ (d) $\log\left|\frac{1+xe^x}{xe^x}\right| - \frac{1}{1+xe^x}$

(93) $\int \frac{dx}{e^x+e^{-x}+2} =$ ______________ $+c$

(a) $-\frac{1}{e^x+1}$ (b) $\frac{1}{e^x+1}$ (c) $-\frac{2^x}{e^x+1}$ (d) $\frac{e^x}{e^x+1}$

(94) If $\int \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}\frac{dx}{x} = k\log\left|\frac{1+\sqrt{1-x}}{\sqrt{x}}\right| - \cos^{-1}\sqrt{x}+c$

(a) 1 (b) 2 (c) -1 (d) -2

(95) $\int \frac{(x+2)^2}{(x+4)}e^x dx =$ ______________ $+c$

(a) $e^x\left(\frac{x}{x+4}\right)$ (b) $e^x\left(\frac{x+2}{x+4}\right)$ (c) $e^x\left(\frac{x-2}{x-4}\right)$ (d) $\frac{2xe^2}{x+4}$

(96) If $\int \frac{2e^x+3e^{-x}}{3e^x+4e^{-x}}dx = Ax + B\log\left|3e^{2x}+4\right| + c$ then $A+B=$ ________

(a) $\frac{11}{24}$ (b) $\frac{13}{24}$ (c) $\frac{15}{24}$ (d) $\frac{17}{24}$

(97) If $\int \frac{dx}{1+\tan^4 x} = k\log\left|\frac{\sec^2 x-\sqrt{2}\tan x}{\sec^2 x+\sqrt{2}\tan x}\right| + \frac{x}{2} + c$ then $k=$ ________

(a) $\frac{1}{4\sqrt{2}}$ (b) $-\frac{1}{4\sqrt{2}}$ (c) $\frac{1}{2\sqrt{2}}$ (d) $-\frac{1}{2\sqrt{2}}$

(98) If $\int \frac{3^x-1}{3^x+1}dx = k\log\left|3^{x/2}+3^{-x/2}\right| + c$ then $k=$ ________

(a) $\log_3^e$ (b) $\log_e^3$ (c) $2\log_3^e$ (d) $2\log_e^3$

(99) $\int \frac{\sin 2x}{\sin^4 x+\cos^4 x}dx =$ __________ $+c$

(a) $\tan^{-1}\left(\sqrt{\tan x}\right)$ (b) $\tan^{-1}\left(\frac{1}{2}\tan x\right)$ (c) $\tan^{-1}\left(\tan^2 x\right)$ (d) $\tan^{-1}\left(2\tan x\right)$

(100) $\int e^{2x}\left(1+\tan x\right)^2 dx =$ ________ $+c$

(a) $\tan e^x$ (b) $\tan xe^{2x}$ (c) $\tan\frac{x}{2}e^x$ (d) $\tan\frac{x}{2}e^{-x}$

(101) $\int \frac{2x^{12}+8x^9}{(x^5+x^3+1)^2}dx =$ ______________ $+c$

(a) $\frac{x^{10}+x^5}{(x^5+x^3+1)^2}$ (b) $\frac{x^5-x^{10}}{(x^5+x^3+1)^2}$ (c) $\frac{x^{10}}{2(x^5+x^3+1)^2}$ (d) $\frac{x^5}{2(x^5+x^3+1)^2}$

(102) $\int \frac{1}{\tan x+\cot x+\sec x+\operatorname{cosec} x}dx =$ ______________ $+c$

(a) $\frac{1}{2}(\cos x-\sin x)+\frac{x}{2}$ (b) $\frac{1}{2}(\sin x-\cos x)-\frac{x}{2}$

(c) $\frac{1}{2}(\sin x+\cos x)+\frac{x}{2}$ (d) $\frac{1}{2}(\sin x+\cos x)-\frac{x}{2}$

(103) $\int \frac{\sec^{\frac{3}{2}}\theta - \sec^{\frac{1}{2}}\theta}{2+\tan^2\theta}\tan\theta\, d\theta =$ _____________ $+c$

(a) $\frac{1}{\sqrt{2}}\log_e\left|\frac{\sec\theta-\sqrt{2\sec\theta}+1}{\sec\theta+\sqrt{2\sec\theta}+1}\right|$ (b) $\frac{1}{\sqrt{2}}\log_e\left|\frac{\sec\theta+\sqrt{2\sec\theta}+1}{\sec\theta-\sqrt{2\sec\theta}+1}\right|$

(c) $\frac{1}{\sqrt{2}}\log_e\left|\frac{\sec\theta-\sqrt{2\sec\theta}-1}{\sec\theta+\sqrt{2\sec\theta}-1}\right|$ (d) $\frac{1}{\sqrt{2}}\log_e\left|\frac{\sec\theta+\sqrt{2\sec\theta}-1}{\sec\theta-\sqrt{2\sec\theta}-1}\right|$

(104) $\int \frac{\sec^2 x - 2009}{\sin^{2009} x}dx =$ _____________ $+c$

(a) $\frac{\cot x}{\sin^{2009} x}$ (b) $\frac{-\cot x}{\sin^{2009} x}$ (c) $\frac{\tan x}{\sin^{2009} x}$ (d) $\frac{-\tan x}{\sin^{2009} x}$

(105) $\int x^{27}(1+x+x^2)^6(6x^2+5x+4)dx =$ _____________ $+c$

(a) $\frac{(x^4+x^3+x^2)^7}{7}$ (b) $\frac{(x^4+x^5+x^6)^7}{7}$

(c) $\frac{(x+x^3+x^5)^7}{7}$ (d) $\frac{(x^5+x^6+x^7)^7}{7}$

(106) $\int \frac{1}{x^2(x^4+1)^{3/4}}dx =$ _____________ $+c$

(a) $\left(1+\frac{1}{x^4}\right)^{1/4}$ (b) $(x^4+1)^{1/4}$ (c) $\left(1-\frac{1}{x^4}\right)^{1/4}$ (d) $-\left(1+\frac{1}{x^4}\right)^{1/4}$

(107) $\int \frac{dx}{x^4+x^3} = \frac{A}{x^2}+\frac{B}{x}+\log\left|\frac{x}{x+1}\right|+c$

(a) $A=\frac{1}{2}, B=1$ (b) $A=1, B=\frac{1}{2}$ (c) $A=-\frac{1}{2}, B=1$ (d) $A=-1, B=-\frac{1}{2}$

(108) $\int \sin x \cdot \cos x \cdot \cos 2x \cdot \cos 4x \cdot \cos 8x \cdot \cos 16x\, dx =$ ____________ $+ c$

(a) $\frac{\sin 16x}{1024}$ (b) $-\frac{\cos 32x}{1024}$ (c) $\frac{\cos 32x}{1096}$ (d) $-\frac{\cos 32x}{1096}$

(109) $\int \frac{\sin x + \sin^3 x}{\cos 2x} dx = A\cos x + B\log|f(x)| + c$

(a) $A = \frac{1}{4}, B = \frac{1}{\sqrt{2}}, f(x) = \frac{\sqrt{2}\cos x - 1}{\sqrt{2}\cos x + 1}$ (b) $A = -\frac{1}{2}, B = \frac{-3}{4\sqrt{2}}, f(x) = \frac{\sqrt{2}\cos x - 1}{\sqrt{2}\cos x + 1}$

(c) $A = -\frac{1}{2}, B = \frac{3}{\sqrt{2}}, f(x) = \frac{\sqrt{2}\cos x + 1}{\sqrt{2}\cos x - 1}$ (d) $A = \frac{1}{2}, B = \frac{-3}{4\sqrt{2}}, f(x) = \frac{\sqrt{2}\cos x - 1}{\sqrt{2}\cos x + 1}$

(110) $\int \frac{x^4 + 1}{x(x^2+1)^2} dx = A\log|x| + \frac{B}{1+x^2} + c.$ *then* $A =$ _____, $B =$ _____

(a) $A = 1; B = -1$ (b) $A = -1; B = 1$ (c) $A = 1; B = 1$ (d) $A = -1; B = -1$

Hints
(Indefinite Integration)

1. $\dfrac{1}{1+\tan x} = \dfrac{\cos x}{\sin x + \cos x} = \dfrac{1}{2}\left[\dfrac{\cos x + \sin x + \cos x - \sin x}{\cos x + \sin x}\right]$

2. $\dfrac{e^x+1}{e^x-1} = \dfrac{e^{\frac{x}{2}} + e^{-\frac{x}{2}}}{e^{\frac{x}{2}} - e^{\frac{-x}{2}}}$ and taking $e^{\frac{x}{2}} - e^{-\frac{x}{2}} = t$

3. $\dfrac{e^{5\log x} - e^{3\log x}}{e^{4\log x} - e^{2\log x}} = \dfrac{x^5 - x^3}{x^4 - x^2} = x$

4. $\dfrac{1}{x\left(x^n+1\right)} = \dfrac{x^{n-1}}{x^n\left(x^n+1\right)}$, taking $x^n = t$

5. Taking $\log(x+1) - \log x = t$

6. $e^{\cot^{-1}x}\left[1 - \dfrac{x}{1+x^2}\right] = e^{\tan^{-1}x} - \dfrac{x}{1+x^2}e^{\tan^{-1}x}$ Integrate $e^{\cot^{-1}x}$ by parts

7. $\dfrac{\tan x}{\sqrt{\cos x}} = (\cos x)^{\frac{-3}{2}}\sin x$, taking $\cos x = t$

8. $e^{4\log x}\left(x^5+1\right)^{-1} = \dfrac{x^4}{x^5+1}$, taking $x^5+1=t$

9. $\operatorname{cosec}^3 x = \operatorname{cosec}^2 x\sqrt{1+\cot^2 x}$, taking $\cot x = t$

10. $\dfrac{2^{\frac{1}{x^2}}}{x^3}$, taking $2^{\frac{1}{x^2}} = t$

11. $(x-1)e^{-x} = xe^{-x} - e^{-x}$ Integrate xe^{-x} by parts

12. $\sin(\log x) - \cos(\log x)$, $\log_e x = t \;\therefore\; x = e^t$

13. $(x+4)(x+3)^7 = [x+3+1][x+3]^7$

$= (x+3)^8 + (x+3)^7$

14. $\dfrac{1}{(x+3)\sqrt{x+2}}$, $x+2=t^2$

15. $\int \frac{1}{e^x + 2 + e^{-x}} = \frac{e^x}{(e^x+1)^2}$ taking $e^x = t$

16. $\frac{\cos x}{\sqrt{\sin^2 x + 2\sin x + 1}}$, taking $\sin x = t$

17. $\frac{1}{e^x + 1} = \frac{e^{-x}}{1 + e^{-x}}$, taking $1 + e^{-x} = t$

18. $\sin^8 x - \cos^8 x = \left(1 - 2\sin^2 x \cos^2 x\right) \cos 2x$

19. Let $\log_c^x = t$ then $d\,(\log x) = dt$

20. $\frac{1 + \cos 8x}{\cot 2x - \tan 2x} = \frac{2\cos^2 4x}{\cos^2 2x - \sin^2 2x} \times \sin 2x \cos 2x = \frac{\sin 8x}{2}$

21. $9e^{2x} - 4 = t$

22. $\frac{1}{\sin^6 x + \cos^6 x} = \frac{1}{1 - 3\sin^2 x \cos^2 x} = \frac{4}{4 - 3\sin^2 2x} = \frac{4\sec^2 2x}{4 + \tan^2 2x}$, and taking $\tan 2x = t$

23. $1 - x^{\frac{3}{2}} = t^2$

24. $\frac{\sec x}{\sqrt{\sin(2x + \alpha) + \sin \alpha}} = \frac{\sec x}{\sqrt{2\sin(x+\alpha)\cos x}} = \frac{\sec^2 x}{\sqrt{2\tan x + \cos\alpha + \sin\alpha}}$

and taking $2\tan x \cos\alpha + \sin\alpha = t^2$

25. $\frac{x^4 + 1}{x^6 + 1} = \frac{x^4 - x^2 + 1 + x^2}{x^6 + 1} = \frac{1}{1 + x^2} + \frac{x^2}{x^6 + 1}$, taking $x^3 = t$

26. $\frac{\log_e x - 1}{(\log_e x)^2}$ taking $\log_e x = t$ $\quad \therefore x = e^t$

27. $\frac{e^x}{x} \log\left(e x^x\right) = \frac{e^x}{x}\left[\log_e e + x\log x\right] = e^x\left[\frac{1}{x} + \log x\right]$

28. $x\,\mathrm{cosec}^2 x$, Let $u = x, v = \mathrm{cosec}^2 x$, taking integration by parts

29. $x^6 \log_e x$, Let $u = \log x, v = x^6$, taking integration by parts

30. $\log(\log x) + \frac{1}{\log x}$, taking $\log_e x = t$ $\quad \therefore x = e^t$

31. $\left(\frac{x^2+1}{x^2}\right) e^{\frac{x^2-1}{x}} = \left(1 + \frac{1}{x^2}\right) e^{x - \frac{1}{x}}$ taking $x - \frac{1}{x} = t$

32. $\dfrac{x^2-1}{(x^4+3x^2+1)\tan^{-1}\left(\dfrac{x^2+1}{x}\right)} = \dfrac{1-\dfrac{1}{x^2}}{\left[\left(x+\dfrac{1}{x}\right)^2+1\right]\tan^{-1}\left(x+\dfrac{1}{x}\right)}$, taking $x+\dfrac{1}{x}=t$

33. $\cos x\, d(\sin x) = \cos x \cos x = \cos^2 x = \dfrac{1+\cos 2x}{2}$

34. taking $x e^x = t$

35. $\sin^3 x = \sin^2 x \sin x = \sin x - \sin x \cos^2 x$, taking $\cos x = t$

36. $\dfrac{1}{e^x + e^{-x}} = \dfrac{e^x}{e^{2x}+1}$ taking $e^x = t$

37. $e^{2x+\log x} = e^{2x}.\ x$ taking $u = x$, $v = e^{2x}$ (integration by parts)

38. $\dfrac{x-\sin x}{1-\cos x} = \dfrac{x-2\sin\dfrac{x}{2}\cos\dfrac{x}{2}}{2\sin^2\dfrac{x}{2}} = x.\dfrac{1}{2}\operatorname{cosec}^2\dfrac{x}{2} - \cot\dfrac{x}{2}$, taking integration by parts

39. $\dfrac{5+\log x}{(6+\log x)^2}$, taking $\log_e x = t \therefore\ x = e^t$

40. $\dfrac{1}{5+4\cos x}$, taking $\tan\dfrac{x}{2} = t$

41. $\dfrac{\log x}{x^2}$, $\log_e x = t \Rightarrow x = e^t$, taking integration by parts

42. $\dfrac{\cos x - \sin x}{(\sin x + \cos x)\sqrt{\sin x\cos x + \sin^2 x\cos^2 x}}$

$= \dfrac{\cos^2 x - \sin^2 x}{(\sin x + \cos x)^2\sqrt{\left(\sin x\cos x + \dfrac{1}{2}\right)^2 - \dfrac{1}{4}}}$

$= \dfrac{2\cos 2x}{(1+\sin 2x)\sqrt{(1+\sin 2x)^2 - 1}}$, taking $1+\sin 2x = t$

43. $\dfrac{\cos x}{\sin^3 x + \cos^3 x} = \dfrac{\operatorname{cosec}^2 x \cdot \cot x}{1 + \cot^3 x}$, taking $\cot x = t$

44. $\sqrt{1 + \sec x} = \sqrt{\dfrac{1 + \cos x}{\cos x}}$, taking $\cos \alpha = y$

45. $\sqrt{\tan x} + \sqrt{\cot x} = \dfrac{\sin x + \cos x}{\sqrt{\sin x \cos x}}$

$= \dfrac{(\sin x + \cos x)\sqrt{2}}{\sqrt{1 - (1 - 2\sin x \cos x)}} = \dfrac{\sqrt{2}\,(\sin x + \cos x)}{\sqrt{1 - (\sin x - \cos x)^2}}$

(taking $\sin x - \cos x = t$)

46. $\dfrac{(x^5 - x)^{\frac{1}{5}}}{x^6} = \dfrac{\left(1 - \frac{1}{x^4}\right)^{\frac{1}{5}}}{x^5}$, taking $1 - \dfrac{1}{x^4} = t$

47. $\dfrac{1}{(x-1)^{\frac{3}{2}}(x-2)^{\frac{1}{2}}} = \dfrac{1}{\left(\frac{x-1}{x-2}\right)^{\frac{3}{2}}(x-2)^2}$, taking $\dfrac{x-1}{x-2} = t$

48. $\dfrac{x^2 + 1}{x^4 - x^2 + 1} = \dfrac{1 + \frac{1}{x^2}}{\left(x - \frac{1}{x}\right)^2 + 1}$, taking $x - \dfrac{1}{x} = t$

49. $\sqrt{\dfrac{\sin x - \sin^3 x}{1 - \sin^3 x}} = \dfrac{\sqrt{\sin x}\,\cos x}{\sqrt{1 - \left(\sin^{\frac{3}{2}} x\right)^2}}$, taking $\sin^{\frac{3}{2}} x = t$

50. $\cot^{-1}\sqrt{x} = u$ and $v = 1$, taking integration by parts

51. $\dfrac{\log x}{(1 + \log x)^2}$, taking $\log_e x = t \Rightarrow x = e^t$

52. $\dfrac{x^2}{(x^2 + 2)(x^2 + 3)} = \dfrac{3(x^2 + 2) - 2(x^2 + 3)}{(x^2 + 2)(x^2 + 3)} = \dfrac{3}{x^2 + 3} - \dfrac{2}{x^2 + 2}$

53. $\dfrac{1+x}{1+\sqrt[3]{x}} = \dfrac{\left(1+\sqrt[3]{x}\right)\left(1-x^{\frac{1}{3}}+x^{\frac{2}{3}}\right)}{1+\sqrt[3]{x}} = 1-x^{\frac{1}{3}}+2^{\frac{2}{3}}$

54. $\dfrac{1}{\left(1+x^2\right)\sqrt{1-x^2}}$, taking $\dfrac{1-x^2}{1+x^2}=t^2 \Rightarrow x^2=\dfrac{1-t^2}{1+t^2}$, $2xdx=\dfrac{-2t\,dt}{\left(1+t^2\right)^2}$

55. $\dfrac{\cot x}{\sqrt{\cos^4 x+\sin^4 x}} = \dfrac{\cot x.\cos ec^2 x}{\sqrt{1+\cot^4 x}}$, taking $\cot^2 x = y$

56. $e^x\left[\dfrac{1-x}{1+x^2}\right]^2 = e^x\left[\dfrac{1}{1+x^2}-\dfrac{2x}{\left(1+x^2\right)^2}\right]$

57. $\dfrac{1}{\sqrt{\cos^3 x \sin\left(x+\alpha\right)}} = \dfrac{\sec^2 x}{\sqrt{\sin\alpha+\cos\alpha\ \tan x}}$, taking $\sin\alpha+\cos\alpha\ \tan x = t^2$

58. $\dfrac{1}{1-\cos^4 x} = \dfrac{1}{2}\left[\dfrac{1}{1-\cos^2 x}+\dfrac{1}{1+\cos^2 x}\right]$

59. $\dfrac{\sqrt{1-\sin x}}{1+\cos x}\,e^{-\frac{x}{2}}$, taking $-\dfrac{x}{2}=t \Rightarrow x=-2t$

60. $\dfrac{1}{(x+2)^{\frac{12}{13}}(x-5)^{\frac{14}{13}}} = \dfrac{1}{\left(\dfrac{x+2}{x-5}\right)^{\frac{12}{13}}(x-5)^2}$, taking $\dfrac{x+2}{x-5}=t$

61. $\dfrac{x^2}{(x\sin x+\cos x)^2} = \dfrac{x}{\cos x}\cdot\dfrac{x\cos x}{(x\sin x+\cos x)^2}$

$u=\dfrac{x}{\cos x}$, taking $v=\dfrac{x\cos x}{(x\sin x+\cos x)^2}$

62. $\left(1+x-\dfrac{1}{x}\right)e^{x+\frac{1}{x}} = e^{x+\frac{1}{x}}+x\left(1-\dfrac{1}{x^2}\right)e^{x+\frac{1}{x}}$

taking $u=x,\ v=e^{x+\frac{1}{x}}\left(1-\dfrac{1}{x^2}\right)$

63. $\dfrac{5x+3}{\sqrt{x^2+4x+10}} = \dfrac{\frac{5}{2}(2x+4)-7}{\sqrt{x^2+4x+10}}$

64. $(1-\cos x)\,\text{cosec}^2 x = \text{cosec}^2 x - \text{cosec}\,x.\cot x$

65. $\dfrac{1}{(2\sin x+3\cos x)^2} = \dfrac{\sec^2 x}{(2\tan x+3)^2}$, taking $\tan x = t$

66. $f(x) = \dfrac{\cos x}{1+\cos x}$ $\quad \therefore \lim_{n\to\infty} Sn = \dfrac{a}{1-r}$ $\quad a = \cos x,\ r = -\cos x$

67. $\dfrac{e^x}{(e^x+2012)(e^x+2013)}$, taking $e^x = t$

68. $\dfrac{x^{2011}\tan^{-1}x^{2012}}{1+x^{4024}}$, taking $\tan^{-1} x^{2012} = t$

69. $\dfrac{1}{\cos x - \sin x}$

$= \dfrac{1}{\sqrt{2}\sin\left(x+\frac{3\pi}{4}\right)} = \dfrac{1}{\sqrt{2}}\,\text{cosec}\left(x+\dfrac{3\pi}{4}\right)$

70. $\dfrac{\sin x}{\sin(x-\alpha)} = \dfrac{\sin(x-\alpha+\alpha)}{\sin(x-\alpha)} = \cos\alpha + \sin\alpha.\cot(x-\alpha)$

71. $\dfrac{5^x}{\sqrt{(5^x)^2-1}}$ taking $5^x = t$

72. $\sin^{-1}\left(\dfrac{2x}{1+x^2}\right)$, taking $x = \tan\theta$

73. $\dfrac{\sin^{-1}\sqrt{x} - \cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x}} = \dfrac{4}{\pi}\sin^{-1}\sqrt{x} - 1$, taking $\sqrt{x} = \sin\theta$

74. $\sin^{-1}\left(2\tan^{-1}\sqrt{\dfrac{1-x}{1+x}}\right)$, taking $x = \cos 2\theta$

75. $\dfrac{(1+x^n)^{\frac{1}{n}}}{x^{n+2}} = \dfrac{\left(\dfrac{1}{x^n}+1\right)^{\frac{1}{n}}}{x^{n+1}}$, taking $x^{-n}+1=t$

76. $5^{5^{5^x}}5^{5^x}5^x$, *taking* $5^{5^{5^x}}=t$

77. $\sqrt{1+\operatorname{cosec} x} = \sqrt{\dfrac{1+\sin x}{\sin x}}$, taking $\sin x = t^2$

78. $\dfrac{1}{\sqrt{1+\operatorname{cosec}^2 x}} = \dfrac{\sin x}{\sqrt{2-\cos^2 x}}$, taking $\cos x = t$

79. $\dfrac{2^{\sqrt{x}}}{\sqrt{x}}$, taking $x = t^2$

80. $\operatorname{cosec}\left(x-\dfrac{\pi}{6}\right)\operatorname{cosec}\left(x-\dfrac{\pi}{3}\right) = 2\left[\cot\left(x-\dfrac{\pi}{3}\right)-\cot\left(x-\dfrac{\pi}{6}\right)\right]$

81. $\dfrac{1}{(\sin^5 x\cot^7 x)^{\frac{1}{6}}} = \dfrac{\sec^2 x}{(\tan x)^{\frac{5}{6}}}$ taking $\tan x = t$

82. $e^x\left[\dfrac{x^3-x-2}{(x^2+1)^2}\right] = e^x\left[\dfrac{x+1}{x^2+1}+\dfrac{1-2x-x^2}{(x^2+1)^2}\right]$,

$f(x) = \dfrac{x+1}{x^2+1}$ $\qquad f'(x) = \dfrac{1-2x-x^2}{(x^2+1)^2}$

83. $\dfrac{(e^x-1)}{(e^x+1)\sqrt{e^x+1+e^{-x}}} = \dfrac{e^{\frac{x}{2}}-e^{\frac{-x}{2}}}{\left(e^{\frac{x}{2}}+e^{\frac{-x}{2}}\right)\sqrt{\left(e^{\frac{x}{2}}+e^{\frac{-x}{2}}\right)^2-1}}$, taking $e^{\frac{x}{2}}+e^{\frac{-x}{2}}=t$

84. $\dfrac{1}{x^{\frac{1}{5}}\sqrt{5^{\frac{8}{5}}-1}}$, taking $x^{\frac{4}{5}}=t$

85. $(x^{30}+x^{20}+x^{10})(2x^{20}+3x^{10}+6)^{\frac{1}{10}}$

$$= \left(x^{30} + x^{20} + x^{10}\right)\left(2x^{30} + 2x^{20} + 6x^{10}\right)^{\frac{1}{10}}$$

taking $2x^{30} + 3x^{20} + 6x^{10} = t$

86. $\dfrac{1}{\sqrt{(x-4)(7-x)}}$, taking $x - 4 = t^2$

87. $\dfrac{2012\,x + 2013}{2013\,x + 2012}$, Nr = A (Dr)+ B

88. $\dfrac{2\sin x + \cos x}{7\sin x - 5\cos x}$; Nr = A + B (Dr)

89. $\dfrac{\cos 9x + \cos 6x}{2\cos 5x - 1} = \dfrac{2\cos\frac{15x}{2}\cos\frac{3x}{2}}{4\cos^2\frac{5x}{2} - 3} = \dfrac{2\left[4\cos^3\frac{5x}{2} - \cos\frac{5x}{2}\right]\cos\frac{3x}{2}}{4\cos^2\frac{5x}{2} - 3} = 2\cos\frac{5x}{2}\cos\frac{3x}{2}$

90. $\dfrac{1}{(x\tan x + 1)^2} = \dfrac{\cos^2 x}{(x\sin x + \cos x)^2} = \dfrac{\cos x}{x}\cdot\dfrac{x\cos x}{(x\sin x + \cos x)^2}$

taking $u = \dfrac{\cos x}{x}$, taking $v = \dfrac{x\cos x}{(x\sin x + \cos x)^2}$

91. $\sqrt{1 + \sin\frac{x}{4}} = \sqrt{\left(\sin\frac{x}{8} + \cos\frac{x}{8}\right)^2} = \sin\frac{x}{8} + \cos\frac{x}{8}$

92. $\dfrac{x+1}{x\left(1 + xe^x\right)^2} = \dfrac{(x+1)\,e^x}{x\,e^x\left(1 + x\,e^x\right)^2}$, taking $x\,e^x = t$

93. $\dfrac{1}{e^x + e^{-x} + 2} = \dfrac{e^x}{\left(e^x + 1\right)^2}$, taking $e^x = t$

94. $\sqrt{\dfrac{1 - \sqrt{x}}{1 + \sqrt{x}}} \times \dfrac{1}{x}$, taking $x = \cos^2\theta$

95. $\dfrac{(x+2)^2}{(x+4)^2}\,e^x = \left(\dfrac{x\,(x+4)}{(x+4)^2} + \dfrac{4}{(x+4)^2}\right)e^x$

$$= \left(\frac{x}{x+4} + \frac{4}{(x+4)^2} \right) e^x, \; f(x) = \frac{x}{x+4} \text{ and } f(x) = \frac{4}{(x+4)^2}$$

96. $\dfrac{2e^x + 3e^{-x}}{3e^x + 4e^{-x}} = \dfrac{2e^{3x} + 3}{3e^{2x} + 4}$, then taking $Nr = A(Dr) + B$

97. $\dfrac{1}{1+\tan^4 x} = \dfrac{\sec^2 x}{(1+\tan^2 x)(1+\tan^4 x)}$, taking $\tan x = t$

98. $\dfrac{3^x - 1}{3^x + 1} = \dfrac{3^{\frac{x}{2}} - 3^{\frac{-x}{2}}}{3^{\frac{x}{2}} + 3^{\frac{-x}{2}}}$, taking $3^{\frac{x}{2}} + 3^{\frac{-x}{2}} = t$

99. $\dfrac{\sin 2x}{\sin^4 x + \cos^4 x} = \dfrac{2\sin x \cos x}{\sin^4 x + \cos^4 x} = \dfrac{2\tan x \cdot \sec^2 x}{1 + \tan^4 x}$, taking $\tan^2 x = t$

100. $e^{2x}(1 + \tan x)^2 = e^{2x}(\tan x + \sec^2 x)$, taking $2x = t$

101 to **110** Try yourself

Answer Key							
1	d	30	d	59	b	88	b
2	a	31	a	60	a	89	b
3	d	32	a	61	a	90	a
4	b	33	c	62	d	91	a
5	c	34	c	63	b	92	c
6	c	35	a	64	a	93	a
7	a	36	b	65	c	94	d
8	d	37	a	66	c	95	a
9	a	38	b	67	a	96	d
10	a	39	b	68	c	97	b
11	b	40	d	69	b	98	c
12	c	41	a	70	a	99	c
13	b	42	a	71	b	100	b
14	a	43	b	72	c	101	c
15	b	44	b	73	b	102	b
16	a	45	a	74	c	103	a
17	a	46	a	75	c	104	c
18	d	47	c	76	c	105	b
19	b	48	b	77	c	106	d
20	c	49	a	78	c	107	c
21	c	50	a	79	c	108	b
22	c	51	a	80	b	109	d
23	c	52	b	81	b	110	c
24	b	53	b	82	c		
25	b	54	a	83	d		
26	c	55	b	84	a		
27	b	56	b	85	c		
28	c	57	a	86	a		
29	a	58	a	87	b		

QUESTION BANK

(Definite Integration)

(1) $\int_{-K}^{K} |x| dx = \frac{1}{K}$; where K $\in$ N then K is ....

(a) 0 (b) 1 (c) 2 (d) not possible

(2) If $\int_{-1}^{n} x|x| dx = \frac{7}{3}, n \in N$ then n is ....

(a) 1 (b) 2 (c) 0 (d) 3

(3) $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} [\cot x] dx$ is equal to ....

(a) 1 (b) 0 (c) $\frac{\pi}{8}$ (d) $\frac{\pi}{4}$

(4) $\int_{0}^{\frac{3}{2}} [x^2] dx$ is equal to ....

(a) $\frac{3}{4}$ (b) 3 (c) $2+\sqrt{2}$ (d) $2-\sqrt{2}$

(5) $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sqrt{1-\sin 2x} dx$ is equal to ....

(a) $\sqrt{2}+1$ (b) $\sqrt{2}-1$ (c) $1-\sqrt{2}$ (d) 0

(6) The value of the integral $\int_{0}^{1} 2^{2x} \cdot 3^{-x} dx$ is ....

(a) $\log_e \frac{64}{27}$ (b) $\log_e \frac{27}{64}$ (c) $\log_{\frac{3}{4}} e$ (d) $\log_{\frac{64}{27}} e$

(7) The value of the integral $\int_{-5}^{5} (x-[x]) dx$ is ....

(a) 0 (b) 5 (c) 10 (d) 15

(8) $\int_{0}^{\frac{\pi}{2}} e^{\sin^{-1} x} \cdot e^{\tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)} dx$ is equal to ....

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{2} e^{\frac{\pi}{2}}$ (c) $\frac{\pi}{4} e^{\frac{\pi}{2}}$ (d) $e^{\frac{\pi}{2}}$

(9) The value of the integral $\int_0^{\frac{\pi}{2}} \left[\tan^{-1}(\cot x) + \cot^{-1}(\tan x)\right] dx$ is ....

(a) $\frac{\pi}{4}$ (b) π (c) $\frac{\pi^2}{4}$ (d) $\frac{\pi^2}{2}$

(10) The value of the integral $\int_{-\pi}^{\pi} \frac{\cos^2 x}{1+3^x} dx$ is ....

(a) 0 (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{2}$ (d) π

(11) The value of the integral $\int_{-1}^{1} \log\left(\frac{1}{x+\sqrt{x^2+1}}\right) dx$ is ....

(a) log 2 (b) 0 (c) log 3 (d) not possible

(12) The value of the integral $\int_0^{e} \frac{x}{\left(x+\sqrt{e^2-x^2}\right)\sqrt{e^2-x^2}} dx$ is ....

(a) 0 (b) $\frac{e}{2}$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$

(13) $\int_0^{2\pi} \left(\sin x + |\sin x|\right) dx$ is equal to ....

(a) 0 (b) 2 (c) -2 (d) 4

(14) $\int_0^{\frac{\pi}{9}} \left(\tan x + \tan 2x + \tan 3x + \tan x \cdot \tan 2x \cdot \tan 3x\right) dx$ is equal to ....

(a) $\frac{1}{3}\log 2$ (b) $\log \sqrt[3]{4}$ (c) $3\log 2$ (d) $4\log\sqrt{3}$

(15) $\int_1^{e} \left(x^x + \log x^{x^x}\right) dx$ is equal to ....

(a) $\frac{e-1}{2}$ (b) $e^e - 1$ (c) $e^e + 1$ (d) e^e

(16) $I = \int_{-1}^{1} \left(x^7 + \cos^{-1} x\right) dx$ then $\cos I$ is equal to ....

(a) 1 (b) 0 (c) -1 (d) $\frac{1}{2}$

(17) $\int_{-\pi/2}^{\pi/2} \sqrt{\cos x - \cos^3 x}\, dx$ is equal to ....

(a) $-\frac{1}{3}$ (b) $-\frac{1}{4}$ (c) $-\frac{2}{3}$ (d) $-\frac{4}{3}$

(18) $\int_{-a}^{a}\left(\frac{|x+a|}{x+a}+\frac{|x-a|}{x-a}\right)dx$ is equal to .... (where a > 0)

(a) 0 (b) a (c) 2a (d) 4a

(19) The value of the integral $\int_{1}^{e}(\log x)^{8}\,dx+8\int_{1}^{e}(\log x)^{7}\,dx$ is ....

(a) e-1 (b) $\frac{\text{e-1}}{2}$ (c) 0 (d) e

(20) If $\int_{\sqrt{2}}^{2}\frac{Kdx}{\sqrt{x^4-x^2}}dx=\frac{\pi}{4}$ then K is equal to ....

(a) 1 (b) 2 (c) 3 (d) 4

(21) $\int_{\log\frac{1}{3}}^{\log 3} 2^{x^2}\cdot x^3 dx$ is equal to ....

(a) 0 (b) log 3 (c) -log3 (d) log2

(22) If f is an even function and $\int_{0}^{2} f(x)dx=K$

then $\int_{-1}^{1}\left(\frac{x^2-1}{x^2}\right)f\left(x+\frac{1}{x}\right)dx$ is equal to ....

(a) 0 (b) 2K (c) K (d) 4K

(23) The value of $\int_{\pi/6}^{\pi/3}\operatorname{cosec}2\theta\log\tan^2\theta\,d\theta$ is ....

(a) 0 (b) $\sqrt{3}$ (c) $\frac{1}{\sqrt{3}}$ (d) 1

(24) The value of $\int_{\frac{\pi}{12}}^{\frac{5\pi}{12}}\frac{dx}{1+\sqrt[n]{\tan x}}=\alpha$ then $\tan\alpha$ is equal to ....

(a) $\sqrt{3}$ (b) 1 (c) $\frac{1}{\sqrt{3}}$ (d) $\frac{\sqrt{3}-1}{2\sqrt{2}}$

(25) The value of $\int_{0}^{\pi/4}\frac{8\tan^2 x+8\tan x+8}{\tan^2 x+2\tan x+1}dx$ is ....

(a) 0 (b) π (c) $\pi+2$ (d) π-2

(26) The value of $\int_{0}^{\pi}\frac{\cos 3\theta}{\cos\theta+\sin\theta}d\theta$ is ....

(a) 0 (b) $\frac{\pi}{2}$ (c) π (d) 2π

(27) $\int_0^{\frac{\pi}{2}} \log\left(\tan\frac{x}{2}+\cot\frac{x}{2}\right)dx$ is equal to ....

(a) $\frac{\pi}{2}\log 2$ (b) $-\frac{\pi}{2}\log 2$ (c) $\pi\log 2$ (d) $-\pi\log 2$

(28) The value of integral $\int_0^{\pi} \frac{\sin(2n+1)\frac{x}{2}}{\sin\frac{x}{2}}dx$ is ....

(a) 0 (b) $\frac{\pi}{2}$ (c) π (d) 2π

(29) The value of the integral $\int_0^{100\pi} \sqrt{1-\cos 2x}\,dx$ is ....

(a) 50π (b) 100π (c) $100\sqrt{2}$ (d) $200\sqrt{2}$

(30) The value of $\int_e^{e^2} \frac{dx}{\log x} - \int_1^2 \frac{e^x}{x}dx$ is ....

(a) e^2 (b) e (c) $\frac{1}{e}$ (d) 0

(31) If $f(x)$ is an odd periodic function with period P then $\int_{2p-a}^{2p+a} f(x)dx$ is equal to ....

(a) p (b) 2p (c) 4p (d) 0

(32) If $I_n = \int_0^1 x^n \cdot e^x dx$ for $n \in N$ then $I_{100} + 100 I_{99}$ is equal to ....

(a) 0 (b) 1 (c) e (d) e^{-1}

(33) $\int_0^{\pi/2} \frac{x\sin x\cos x}{\cos^4 x+\sin^4 x}dx$ is equal to ....

(a) $\frac{\pi}{4}$ (b) $\frac{\pi^2}{16}$ (c) $\frac{\pi^2}{4}$ (d) $\frac{\pi}{16}$

(34) $\int_0^{\pi/2} \log\left(\frac{a+b\sin x}{a+b\cos x}\right)dx$ is equal to ....

(a) 0 (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{4}$ (d) πab

(35) The value of intergral $\int_1^2 \frac{dx}{x+x^7}$ is ....

(a) $\frac{1}{6}\log\frac{64}{65}$ (b) $\frac{1}{6}\log\frac{128}{65}$ (c) $\frac{1}{6}\log\frac{32}{65}$ (d) $6\log\frac{64}{65}$

(36) If $I_n = \int_0^{\pi/4} \tan^n x dx$ then $\sum_{r=1}^{5} \frac{1}{I_r + I_{r+2}}$ is equal to ....

(a) 5 (b) 10 (c) 15 (d) 20

(37) If $\int_{3+\pi}^{4+\pi} f(x-\pi)dx = \int_a^b f(x)dx$ then $a+b$ is equal to ....

(a) $2\pi+7$ (b) $\pi+\frac{7}{2}$ (c) $\frac{1}{2}$ (d) 7

(38) $\int_0^1 \sqrt[3]{x^3 - x^4}\, dx$ is equal to ....

(a) $\frac{1}{2}$ (b) $\frac{3}{7}$ (c) $\frac{9}{28}$ (d) $\frac{29}{28}$

(39) The value of the integral $\int_0^1 \left(x^5+6x^4+5x^3+4x^2+3x+1\right)e^{x-1} dx$ is equal to ....

(a) 5 (b) $5e$ (c) $5e^2$ (d) $5e^4$

(40) $\int_0^2 x^{[x]} dx$ is equal to ....

(a) $\frac{1}{2}$ (b) $\frac{3}{2}$ (c) $\frac{5}{2}$ (d) $\frac{7}{2}$

(41) If $f(x) = f(\pi+e-x)$ and $\int_e^{\pi} f(x)dx = \frac{2}{e+\pi}$ then $\int_e^{\pi} xf(x)\,dx$ is equal to ....

(a) $\frac{\pi+e}{2}$ (b) $\frac{\pi-e}{2}$ (c) 1 (d) -1

(42) The value of integral $\int_0^1 \frac{1}{1-x+\sqrt{2x-x^2}} dx$ is ....

(a) 1 (b) $\frac{1}{2}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{2}$

(43) If $\int_{\frac{\pi}{4}}^{\pi/2} \frac{\cos^n x}{\sin^{n+2} x} dx = \frac{1}{K-1}$ then K is equal to ....

(a) n (b) $n+1$ (c) $n+2$ (d) $n+3$

(44) $\int_0^{\pi/4} \log(\cot 2x)^{\sin 4x} dx$ is equal to ....

(a) 0 (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{8}$ (d) $\frac{\pi}{2}$

(45) If $\int_n^{n+1} f(x)dx = n$ where $n = 0,1,2......$, and $\int_0^{100} f(x)dx = \frac{k^2 - k}{2}$ then k is

(a) 50 (b) 49 (c) 99 (d) 100

(46) $\int_{-\pi}^{\pi} \frac{2x(1+\sin x)}{1+\cos^2 x} dx$ is equal to ....

(a) 0 (b) π (c) $\frac{\pi^2}{2}$ (d) π^2

(47) The value of integral $\int_a^{a+1} |a - x| dx$ is $(a \in R^+) =$ ....

(a) a (b) $\frac{a}{2}$ (c) 1 (d) $\frac{1}{2}$

(48) The value of $\int_0^{\frac{\pi}{2}} \sin\theta\sqrt{\sin 2\theta}\, d\theta$ is ....

(a) 1 (b) 0 (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$

(49) $\int_{e^{-1}}^{1} \left|\log x^{\frac{1}{x}}\right| dx$ is equal to

(a) $\frac{1+e}{2}$ (b) $\frac{e-1}{2}$ (c) 1 (d) $\frac{1}{2}$

(50) If $a < 0 < b$ then the value of $\int_a^b \frac{|x|}{x} dx$ is ....

(a) $a+b$ (b) $b-a$ (c) $a-b$ (d) $\frac{b-a}{2}$

(51) $\int_0^{\pi/2} \sqrt{\sec x + 1}\, dx$ is equal to ....

(a) 0 (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{2}$ (d) π

(52) The value of the integral $\int_{-\pi/4}^{\pi/4} \log(\sec\theta - \tan\theta)\, d\theta$ is ....

(a) $\pi/4$ (b) $\pi/2$ (c) π (d) 0

(53) The value of the integral $\int_0^{\pi} \sqrt{\sin x} \cdot \cos\frac{x}{2}\, dx$ is ....

(a) 0 (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{2}$ (d) π

(54) $\int_0^1 \sqrt{x}\sqrt{1-\sqrt{x}}\, dx$ is equal to ....

(a) $\frac{4}{105}$ (b) $\frac{8}{105}$ (c) $\frac{16}{105}$ (d) $\frac{32}{105}$

(55) $\int_0^{\pi/4} \frac{\sin 2\theta}{\cos^4\theta + \sin^4\theta}\, d\theta$ is equal to ....

(a) 0 (b) $\frac{\pi}{8}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{2}$

(56) $\int_0^{\pi} \frac{\sin 100x}{\sin x}\, dx$ is equal to ....

(a) 0 (b) π (c) $\frac{\pi}{2}$ (d) 2π

(57) $\int_{-\pi/2}^{\pi/2} \sin x f(\cos x)\, dx$ is equal to ....

(a) 1 (b) -1 (c) 0 (d) $\frac{\pi}{2}$

(58) The value of the integral $\int_{-1}^{1} (x^2 + x)|x|\, dx$ is ....

(a) 0 (b) $\frac{1}{2}$ (c) 1 (d) 2

(59) The value of $\int_0^{\pi} [\cot x]\, dx$ is equal to .... (where [ ] denotes the greatest integer function)

(a) $\frac{\pi}{2}$ (b) 1 (c) $\frac{-\pi}{2}$ (d) -1

(60) If $f(x)=\int_0^x log\left(\frac{1-t}{1+t}\right)dt$ then $f\left(\frac{1}{2}\right)-f\left(\frac{-1}{2}\right)$ is equals to ....

(a) 0 (b) $\frac{1}{2}$ (c) $\frac{-1}{2}$ (d) 1

(61) The value of $c\int_{1+c}^{a+c}\left[f(cx)+1\right]dx-\int_c^{ac} f\left(c^2+x\right)dx, c\neq 0,$ is equal to ....

(a) 0 (b) $c(a-1)$ (c) ac (d) $a(c+1)$

(62) $f:R\rightarrow R$ and satisfies $f(2)=-1, f'(2)=4$ If $\int_2^3 (3\text{-x})\, f''(x)dx=7,$

then $f(3)$ is equal to ....

(a) 2 (b) 4 (c) 8 (d) 10

(63) $\int_1^{\frac{1}{e}}\frac{log\, t}{1+t}dt+\int_1^{e}\frac{log\, t}{1+t}dt$ is equal to ....

(a) e (b) $\frac{1}{e}$ (c) 2 (d) $\frac{1}{2}$

(64) If $\int_0^{\pi} f(sin\, x)\, dx=2$ then the value of $\int_0^{\pi} xf(sin\, x)\, dx$ is ....

(a) 0 (b) 4 (c) $\frac{\pi}{2}$ (d) π

(65) $\int_{-3\pi/2}^{-\pi/2}\left[(x+\pi)^3+cos^2(x+3\pi)\right]dx$ is equal to ....

(a) $\frac{\pi^3}{8}$ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{4}$-1 (d) $\frac{\pi}{4}$+1

(66) If $f(x)=1-\frac{1}{x}$ then $\int_{1/3}^{2/3} fof(x)\, dx$ is equal to ....

(a) 1 (b) $\frac{1}{2}$ (c) -log2 (d) -log$\frac{1}{2}$

(67) The value of the integral $\int_0^1\frac{dx}{x^{3/2}+x^{1/2}}$ is ....

(a) 0 (b) 1 (c) $\frac{\pi}{2}$ (d) π

(68) The value of the integral $\int_0^{1/2}\frac{dx}{(1-x)^{3/2}\sqrt{1+x}}$ is ....

(a) 0 (b) $\frac{1}{2}$ (c) 1 (d) 2

(69) If $h(x)=\left[f(x)+g(x)\right]\left[g(x)-f(x)\right]$ where f is an odd and g is an even function the $\int_{-\pi/2}^{\pi/2} h(x)\,dx$ is equal to ....

(a) 0 (b) $\frac{\pi}{2}$ (c) $\int_0^{\pi/2} h(x)\,dx$ (d) $2\int_0^{\pi/2} h(x)\,dx$

(70) If $\int_0^{100} f(x)dx=10$ then $\sum_{K=1}^{100}\int_0^1 f(x+K-1)\,dx$ is equal to ....

(a) 0 (b) 10 (c) 100 (d) 1000

(71) If f is an odd function the value of integral $\int_{1/e}^{e} \frac{1}{x} f\left(x-\frac{1}{x}\right)dx$ is equal to ....

(a) e (b) $\frac{e^2+1}{e}$ (c) $\frac{e^2-1}{2e}$ (d) 0

(72) The value of the integral $\int_0^{\pi/2} \sin\theta\cdot\log\sin\theta\cdot d\theta$ is ....

(a) $\log\frac{2}{e}$ (b) $\log 2e$ (c) $\log 2$ (d) $\log\frac{e}{2}$

(73) The value of the integral $\int_0^1 \log\left(\frac{1}{x}-1\right)dx$ is ....

(a) 1 (b) $\frac{1}{2}$ (c) 0 (d) 2

(74) The value of integral $\int_0^{\frac{\pi}{4}} \frac{2}{\sec x+\operatorname{cosec} x+\tan x+\cot x}dx$ is ....

(a) 0 (b) 1-$\frac{\pi}{4}$ (c) $\frac{\pi}{4}$+1 (d) $\frac{\pi}{2}$+1

(75) The value of the integral $\int_{\frac{\sqrt{5}+1}{2}}^{\sqrt{3}} \frac{(x^2+1)\,dx}{(x^2-1)\sqrt{x^4-3x^2+1}}$ is ....

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{12}$ (d) $\frac{\pi}{4}$

(76) The value of the integral $\int_0^1 \log\left(\sqrt{1-x}+\sqrt{1+x}\right)dx$ is ....

(a) $\frac{1}{2}\left[log(2)-\frac{1}{2}+\frac{\pi}{4}\right]$ (b) $\frac{1}{2}\left(log\,2-1+\frac{\pi}{2}\right)$

(c) $\frac{1}{3}\left(log\,4-1+\frac{\pi}{4}\right)$ (d) $\frac{1}{4}\left(log\,3-1+\frac{\pi}{2}\right)$

(77) The area enclosed by the parabola $x^2=4by$ and its latusrectum is $\frac{8}{3}$ then $b>0$ is equal to ....

(a) 2 (b) $\sqrt{2}$ (c) 1 (d) 4

(78) The parabolas $y^2=4x$ and $x^2=4y$ divide the square region bounded the lines $x=4, y=4$ and the coordinate axes. If S_1, S_2, S_3 are respectively the areas of these parts numered from top to bottom then $S_1 : S_2 : S_3$ is ....

(a) 1:2:3 (b) 2:1:2 (c) 3:2:3 (d) 1:1:1

(79) The area enclosed between the curves $y=log_e(x+e)$ and the coordinate axes is ....

(a) 1 (b) 4 (c) 2 (d) 3

(80) Ratio of the area cut off by a parabola $y^2=32x$ and line $x=8$ corresponding rectangle contained the area formed by above curves region is ....

(a) $\frac{3}{2}$ (b) $\frac{2}{3}$ (c) $\frac{1}{3}$ (d) 3

(81) The area bounded by $|x|-|y|=2$ is ....

(a) 2 Sq. unit (b) 4 Sq. unit (c) 8 Sq. unit (d) 16 Sq. unit

(82) The area bounded by the curves $x^2=y$ and $2x+y-8=0$ and $y-axis$ in the second quadrant is ....

(a) 9 Sq. unit (b) 18 Sq. unit (c) $\frac{80}{3}$ Sq. unit (d) 36 Sq. unit

(83) The area of common region of the circle $x^2+y^2=4$ and $x^2+(y-2)^2=4$ is ....

(a) $\frac{1}{3}(4\pi-2\sqrt{3})$ (b) $\frac{4}{3}(2\pi-\sqrt{3})$ (c) $\frac{4}{3}(\sqrt{3}-2\pi)$ (d) $\frac{2}{3}(4\pi-3\sqrt{3})$

(84) The area enclosed between the curves $y=kx^2$ and $x^2=ky^2\ (k>0)$ is 12 Sq. unit Then the value of 'k' is ....

(a) 6 (b) $\frac{1}{6}$ (c) 12 (d) $\frac{1}{12}$

(85) The area enclosed by $y^2 = 32x$ and $y = mx(m > 0)$ is $\frac{8}{3}$ then m is ....

(a) 1 (b) 2 (c) 4 (d) $\frac{1}{4}$

(86) The area of the region bounded by the circle $x^2 + y^2 = 12$ and parabola $x^2 = y$ is ....

(a) $(2\pi - \sqrt{3})$ Sq.unit (b) $4\pi + \sqrt{3}$ Sq.unit

(c) $2\pi + \sqrt{3}$ Sq.unit (d) $\pi + \frac{\sqrt{3}}{2}$ Sq.unit

(87) The area bounded by the curves $|x| + |y| \geq 2$ and $x^2 + y^2 \leq 4$ is ....

(a) 4π-4 (b) 4π-2 (c) $4(\pi$-2) (d) $4(\pi$-1)

(88) The area bounded by the curves $y = x^2$ and $y = |x|$ is ....

(a) 1 Sq.unit (b) 2 Sq.unit (c) $\frac{1}{3}$ Sq.unit (d) $\frac{2}{3}$ Sq.unit

(89) The area of the region bounded by curves $f(x) = \sin x, g(x) = \cos x, x = \frac{\pi}{4}, x = \frac{5\pi}{4}$ is ....

(a) 1 (b) 2 (c) $\sqrt{2}$ (d) $2\sqrt{2}$

(90) The area enclosed by the curves $x^2 = y, y = x + 2$ and $x-$axis is ....

(a) $\frac{3}{2}$ (b) $\frac{5}{2}$ (c) $\frac{5}{6}$ (d) $\frac{7}{6}$

(91) The area bounded by ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$ and its auxiallary circle is ....

(a) 2π (b) 3π (c) 6π (d) 9π

(92) The area of the region bounded by curves $x^2 + y^2 = 4, x = 1 \& x = \sqrt{3}$ is.........................

(a) $\frac{\pi}{3}$ sq.unit (b) $\frac{2\pi}{3}$ sq.unit (c) $\frac{s\pi}{6}$ sq.unit (d) $\frac{4\pi}{3}$ sq.unit

(93) The area of the region bounded by the lines $y = mx, x = 1, x = 2$ and $x-$axis is 6 Sq.unit then m is ....

(a) 1 (b) 2 (c) 3 (d) 4

Hints
(Definite Integration)

1. $|x|$ is an even function

$$\therefore \int_{-k}^{k} |x|\ dx = 2\int_{0}^{k} x\,dx = k^2$$

$$\therefore k^2 = \frac{1}{k}$$

2. Here $\int_{-1}^{1} x|x|dx + \int_{1}^{n} x|x|dx$

$$\frac{7}{3} = 0 + \int_{1}^{n} x^2\,dx \qquad (\therefore x|x|) \text{ is an odd function)}$$

$$\frac{7}{3} = \frac{4^3 - 1}{3} \qquad (\therefore x > 0)$$

3. $$I = \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 0\,dx \qquad \left[\begin{array}{l} \because \frac{\pi}{3} \le x \le \frac{\pi}{2} \\ \therefore \frac{1}{\sqrt{3}} > \cot x > 0 \end{array}\right]$$

$$= 0$$

4. $$\int_{0}^{\frac{3}{2}} \left[x^2\right] dx = \int_{0}^{1} \left[x^2\right] dx + \int_{1}^{\sqrt{2}} \left[x^2\right] dx + \int_{\sqrt{2}}^{\frac{3}{2}} \left[x^2\right] dx$$

$$= 0 + \int_{1}^{\sqrt{2}} 1.\,dx + \int_{\sqrt{2}}^{\frac{3}{2}} 2\,dx$$

5. Here $\frac{\pi}{4} < x < \frac{\pi}{2}$

$\therefore \cos x < \sin x$

$$\therefore I = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (\sin x - \cos x)\,dx$$

6. $\int_0^1 \left(\frac{4}{3}\right)^x dx = \left[\frac{\left(4/3\right)^x}{\log_e 4/3}\right]_0^1$

$$= \frac{1}{\log_e 4/3}\left[\frac{4}{3} - 1\right]$$

7. $\int_{-5}^{5} (x - [x])\, dx$

$$= \int_{-5}^{5} x\, dx - \left[\int_{-5}^{-4} [x]\, dx + \int_{-4}^{-3} [x] dx + ...+ \int_{4}^{5} [x]\, dx\right]$$

$$= 0 - [-5 \; -4 - + ...3 + 4]$$

$$= 5$$

8. $\int_0^{\frac{\pi}{2}} \left(e^{\sin^{-1} x} + e^{\cos^{-1} x}\right) dx = e^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} dx$

9. $I = \int_0^{\frac{\pi}{2}} \left[\tan^{-1}\left(\cot\left(\frac{\pi}{2} - x\right)\right) + \cot^{-1}\left(\tan\left(\frac{\pi}{2} - x\right)\right)\right] dx$

$$= \int_0^{\frac{\pi}{2}} (x + x)\, dx$$

10. $I = \int_{-\pi}^{\pi} \frac{\cos^2 x}{1 + 3^x}$ ....(I)

$$I = \int_{-\pi}^{\pi} \frac{\cos^2 (o - x)}{1 + 3^{o-x}}\, dx = \int_{-\pi}^{\pi} \frac{\cos^2 x}{1 + \frac{1}{3^x}}\, dx \qquad ....(II)$$

$$2I = \int_{-\pi}^{\pi} \cos^2 x\, dx$$

11. $f(x) = \log\left(\frac{1}{x+\sqrt{x^2+1}}\right)$

$f(-x) = \log\left(\frac{1}{-x+\sqrt{x^2+1}}\right)$

$= \log\left(x+\sqrt{x^2+1}\right)$

$= -f(x)$

$\therefore \int_{-1}^{1} f(x)dx = 0$

12. $\int_0^e \frac{xdx}{\left(x+\sqrt{e^2-x^2}\right)\sqrt{e^2-x^2}}$

(*Take* $x = e\sin\theta \therefore dx = e\cos\theta d\theta$)

$= \int_0^{\frac{\pi}{2}} \frac{\sin\theta}{\sin\theta+\cos\theta} d\theta = \frac{\pi}{4}$

13. $I = \int_0^{\pi}(\sin x + |\sin x|)dx + \int_{\pi}^{2\pi}(\sin x+|\sin x|)dx$

$= 2\int_0^{\pi}\sin x \, dx + 0$ $\quad (\because \pi < x < 2\pi \Rightarrow \sin x < 0 \;\&\; 0 < x < \pi \Rightarrow \sin x > 0)$

14. $\tan 3x = \frac{\tan 2x + \tan x}{1-\tan x \cdot \tan 2x}$

$\therefore \tan x + \tan 2x + \tan 3x + \tan x \,.\, \tan 2x \,.\, \tan 3x = 2 \tan 3x$

$\therefore I = 2\int_0^{\frac{\pi}{9}} \tan 3x \, dx$

15. $I = \int_1^{e^e} .dt$

Put $x^x = t \quad \therefore \quad x^x(\log x + 1)\,dx = dt$

16. $I = \int_{-1}^{1} x^7\,dx + \int_{-1}^{1}\cos^{-1}x\,dx$

$$= 0 + \int_{-1}^{1} \cos^{-1}\left(1 + (-1) - x\right) dx$$

$$= \int_{-1}^{1}\left(\pi - \cos^{-1} x\right) dx = \int_{-1}^{1} \pi\, dx - I$$

$$2I = \int_{-1}^{1} \pi\, dx$$

17. $I = 2\int_{0}^{\frac{\pi}{2}} \sqrt{\cos x} \cdot |\sin x|\, dx$ (even function)

$= 2\int_{0}^{\frac{\pi}{2}} \sqrt{\cos x} \cdot \sin x\, dx$ $(\because \sin x > 0)$

18. $-a < x < a$

$0 < x + a < 2a \;\&\; -2a < x - a < 0$

$$\therefore\; I = \int_{-a}^{a}\left(\frac{x+a}{x+a} + \frac{a-x}{x-a}\right) dx = 0$$

19. $\int_{1}^{e} (\log x)^8\, dx = \left[x\,(\log x)^8\right]_1^e - 8\int_{1}^{e} x\,(\log x)^7 \cdot \frac{1}{x}\, dx$

$$\therefore \int_{1}^{e} (\log x)^8\, dx + 8\int_{1}^{e} (\log x)^7\, dx = \left[x\,(\log x)^8\right]_1^e$$

20. $\int_{\sqrt{2}}^{2} \frac{k\,dx}{x\sqrt{x^2 - 1}} = \frac{\pi}{4}$

$$k\left[\sec^{-1} x\right]_{\sqrt{2}}^{2} = \frac{\pi}{4}$$

21. $I = \int_{-\log 3}^{\log 3} 2^{x^2} \cdot x^3\, dx$ $(\because f \text{ is an odd function})$

$= 0$

22. $x + \frac{1}{x} = t$

$$\left(1 - \frac{1}{x^2}\right) dx = dt$$

$$\therefore\ I = \int_{-2}^{2} f(t)\,dt = 2\int_{0}^{2} f(t)\,dt \quad (\because f \text{ is an odd function})$$

23. $\log \tan\theta = t$

$$\frac{1}{\tan\theta}\,.\,\sec^2\theta\,.\,d\theta = dt$$

$$I = \frac{1}{2}\int_{-\log\sqrt{3}}^{\log\sqrt{3}} t\,dt = 0$$

24. $$I = \int_{\frac{\pi}{12}}^{\frac{5\pi}{12}} \frac{\sqrt[n]{\cos x}}{\sqrt[n]{\cos x} + \sqrt[n]{\cos x}}\,dx$$

$$I = \int_{\frac{\pi}{12}}^{\frac{5\pi}{12}} \frac{\sqrt[n]{\cos x}}{\sqrt[n]{\cos x} + \sqrt[n]{\sin x}}\,dx$$

$$2I = \int_{\frac{\pi}{12}}^{\frac{5\pi}{12}} 1.dx = \frac{\pi}{3}$$

25. $$I = 4\int_{0}^{\frac{\pi}{4}} \frac{2\tan^2 x + 2\tan x + 2}{\tan^2 x + 2\tan x + 1}\,dx$$

$$= 4\int_{0}^{\frac{\pi}{4}} 1.dx + 4\int_{0}^{\frac{\pi}{4}} \frac{1+\tan^2 x}{(\tan x + 1)^2} \qquad \left(\begin{array}{l}\because\ 1+\tan x = t\\ \sec^2 x\ dx = dt\end{array}\right)$$

$$= \pi + 4\int_{1}^{2} \frac{1}{t^2}.dt$$

26. $$I = \int_{0}^{\pi} \frac{\cos 3\theta}{\cos\theta + \sin\theta}\,d\theta \qquad \text{...(i)}$$

$$I = \int_{0}^{\pi} \frac{\cos(3(\pi-\theta))}{\cos(\pi-\theta) + \sin(\pi-\theta)}\,d\theta \qquad \text{...(ii)}$$

$$(i) + (ii) \Rightarrow 2I = \int_{0}^{\pi} \frac{2\cos 3\theta.\cos\theta}{\cos 2\theta}\,d\theta$$

$$= \int_0^{\pi} \frac{\cos 4\theta + \cos 2\theta}{\cos 2\theta}\, d\theta$$

27. $$I = \int_0^{\frac{\pi}{2}} \log\left(2\frac{\left(1+\tan^2\frac{x}{2}\right)}{2\tan\frac{x}{2}}\right) dx$$

$$= \int_0^{\frac{\pi}{2}} \log 2\, dx - \int_0^{\frac{\pi}{2}} \log \sin x\ dx$$

$$= \frac{\pi}{2}\log 2 - \left(-\frac{\pi}{2}\log 2\right)$$

28. $$\sin(2n+1)\frac{x}{2} = \sin(2n+1)\frac{x}{2} - \sin(2n-1)\frac{x}{2} + \sin(2n-1)\frac{x}{2}$$

$$-\sin(2n-3)\frac{x}{2} + \dots\dots\dots + \sin\frac{3x}{2} - \sin\frac{x}{2} + \sin\frac{x}{2}$$

$$= 2\cos nx.\sin\frac{x}{2} + 2\cos(n-1)\,x.\sin\frac{x}{2} + \dots\dots\dots + 2\cos x\,.$$

$$\sin\frac{x}{2} + \sin\frac{x}{2}$$

$$I = 2\int_0^{\pi}\left(\cos nx + \cos(n-1)x + \dots\dots\dots + \cos x + \frac{1}{2}\right)dx$$

29. $$I = \int_0^{100\pi} \sqrt{2}\,|\sin x|\,dx$$

$$= \sqrt{2}\left[\int_0^{\pi}\sin x\ dx - \int_{\pi}^{2\pi}\sin x\ dx + \int_{2\pi}^{3\pi}\sin x dx + \dots\dots\dots + \int_{98\pi}^{99\pi}\sin x\, dx - \int_{99\pi}^{100\pi}\sin x\ dx\right]$$

30. $$\int_e^{e^2}\frac{dx}{\log x} = \int_1^2 \frac{e^t}{t}\,dt$$ [put $\because \log x = t \quad \therefore\ x = e^t \quad \therefore\ dx = e^t dt$]

$$= \int_1^2 \frac{e^x}{x}dx$$

$$\therefore \int_e^{e^2}\frac{dx}{\log x} - \int_e^2 \frac{e^x}{x}\,dx = 0$$

31. $\int_{2P-a}^{2P+a} f(x)\,dx = \int_{2P-a}^{2P+a} f\,(4P-x)\,dx$

$= -\int_{2p-a}^{2p+a} f\,(x+(-4P))\,dx$ [$\because f(-x) = -f(x)$][$-4p$ is period of f]

$= -\int_{2P-a}^{2P+a} f(x)\,dx$

$= -I$

32. $I_{100} = \int_0^1 x^{100} e^x \,dx$

$= \left[x^{100}e^x\right]_0^1 - \int_0^1 100x^{99}e^x dx$

$= e - 100\,I_{99}$

33. $I = \int_0^{\frac{\pi}{2}} \frac{x\sin x\,\cos x}{\sin^4 x + \cos^4 x}\,dx$

$I = \int_0^{\frac{\pi}{2}} \frac{\left(\frac{\pi}{2} - x\right)\cos x \cdot \sin x}{\cos^4 x + \sin^4 x}\,dx = \int_0^{\frac{\pi}{2}} \frac{\pi}{2}\frac{\cos x \sin x}{\cos^4 x + \sin^4 x} - I$

34. $I = \int_0^{\frac{\pi}{2}} \log\left(\frac{a + b\sin x}{a + b\cos x}\right)dx$

$I = \int_0^{\frac{\pi}{2}} \log\left(\frac{a + b\cos x}{a + b\sin x}\right)dx$

$2I = \int_0^{\frac{\pi}{2}} \log 1\,dx = 0$

35. $\int_1^2 \frac{dx}{x(1+x^6)}$ [Put $t = x^6$, $dt = 6x^5\,dx$]

$= \int_1^{64} \frac{dt}{6t\,(t+1)}$

36. $I_K + I_{K+2} = \int_0^{\frac{\pi}{4}} \tan^k x \left(1 + \tan^2 x\right) dx$

$$= \left[\frac{\tan^{k+1} x}{k+1}\right]_0^{\frac{\pi}{4}} = \frac{1}{k+1}$$

$$\sum_{r=1}^{5} \frac{1}{I_r + I_{r+2}} = \frac{1}{I_1 + I_3} + \text{..........} + \frac{1}{I_5 + I_7} = 2 + 3 + \text{............} + 6$$

$= 20$

37. $\int_{3+\pi}^{4+\pi} f\,(x - \pi)dx$

$= \int_3^4 f\,(t)\,dt$ [Put $x - \pi = t$ $dx = dt$]

$a = 3,\ b = 4$ $a + b = 7$

38. $\int_0^1 \sqrt[3]{x^3 - x^4}$ $= \int_0^1 x\ \sqrt[3]{1 - x}$

$= \int_0^1 (1 - x)\ \sqrt[3]{x}\ dx$

39. $\int_0^1 (x^5 + 5x^4 + x^4 + 4x^3 + x^3 + 3x^2 + x^2 + 2x + x + 1)\ \frac{e^x}{e}\ dx$

$$= \frac{1}{e}\left[x^5 + x^4 + x^3 + x^2 + x\ e^x\right]_0^1$$

40. $I = \int_0^1 x^{[x]}\ dx + \int_1^2 x^{[x]}\ dx$

$= \int_0^1 dx + \int_1^2 x\ dx$

41. $I = \int_e^{\pi} (e + \pi - x)\ f\ (e + \pi - x)\ dx$

$$= \int_e^{\pi} (e + \pi)\, f(x)\, dx - I \quad (\because f(e + \pi - x) = f(x))$$

$$I = \frac{e+\pi}{2} \cdot \frac{2}{e+\pi} = 1$$

42. $$I = \int_0^1 \frac{dx}{1-(1-x)+\sqrt{2(1-x)-(1-x)^2}}$$

$$= \int_0^1 \frac{1}{x+\sqrt{1-x^2}}$$

$$= \int_0^{\pi/2} \frac{\cos\theta}{\sin\theta+\cos\theta}\, dQ \qquad [\because x = \sin\theta,\ dx = \cos\theta \,.\, d\theta]$$

43. $$\frac{1}{k-1} = \int_{\pi/4}^{\pi/2} \cot^n x \,.\, cosec^2 \times dx$$

$$= -\left[\frac{\cot^{n+1} x}{n+1}\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}} = \frac{1}{n+1}$$

44. $$I = \int_0^{\pi/4} \sin 4x \log \cot 2x\, dx$$

$$= \int_0^{\pi/4} \sin\left[\frac{4\pi}{4} - 4x\right] \log \cot\left[\frac{\pi}{2} - 2x\right] dx$$

$$I = \int_0^{\pi/4} \sin 4x \,.\, \log \tan 2x\, dx$$

$$2I = 0$$

45. $$\int_0^{100} f(x)\, dx = \int_0^1 f(x)\, dx + \int_0^2 f(x)\, dx + \dots + \int_{99}^{100} f(x)\, dx$$

$$\frac{k(k-1)}{2} = 0 + 1 + 2 + \dots\dots\dots + 99$$

46. $I = \int_{-\pi}^{\pi} \frac{2x(1+\sin x)}{1+\cos^2 x}\,dx$

$= \int_{-\pi}^{\pi} \frac{2x\,dx}{1+cos^2 x} + \int_{-\pi}^{\pi} \frac{2x\sin x}{1+\cos^2 x}\,dx$

$= 0 + 2\int_{0}^{\pi} \frac{x\sin x}{1+\cos^2 x}\,dx$

47. $\int_{a}^{a+1} |a - x|\,dx$

$= \int_{a}^{a+1} (x - a)\,dx$

$(a < x < a + 1;\ 0 < x - a < 1)$

48. $I = \int_{0}^{\pi/2} \sqrt{\sin 2\theta}\ \sin\theta\,d\theta$ .... (i)

$I = \int_{0}^{\pi/2} \sqrt{\sin(\pi - 2\theta)}\ .\ \sin\left(\frac{\pi}{2} - 0\right)d\theta$

$= \int_{0}^{\pi/2} \sqrt{\sin 2\theta}\ .\cos\theta.d\theta$ ..... (ii)

$2I = \int_{0}^{\pi/2} \sqrt{\sin\alpha\theta}\ (\sin\theta + \cos\theta)\,d\theta$ ($\because$ (i) + (ii))

$= \int_{0}^{\pi/2} \sqrt{1-(\sin\theta - \cos\theta)^2}\ .\ (\sin\theta + \text{coos}\theta)\,d\theta$

put $\sin\theta - \cos\theta = t$

$(\cos\theta + \sin\theta)\,d\theta = dt$

49. $\int_{1/e}^{1} \left|\log x^{\frac{1}{x}}\right| dx$

$= -\int_{1/e}^{1} \frac{1}{x}\log x\,dx$ $\qquad [\because \frac{1}{x} > 0\ \&\ ;\log x < 0]$

$$= -\left[\frac{(\log x)^2}{2}\right]_{\frac{1}{e}}^{1}$$

50. $a < 0 < b$

$$\therefore \int_a^b f(x)\, dx = \int_a^0 \frac{|x|}{x}\, dx + \int_a^b \frac{|x|}{x}\, dx$$

$$= -\int_a^0 1 \,.\, dx + \int_a^b 1 \,.\, dx$$

51. $I = \int_0^{\pi/2} \sqrt{\frac{1+\cos x}{\cos x}}\, dx$

$$= \int_0^{\pi/2} \frac{\sqrt{2}\cos\frac{x}{2}}{\sqrt{1-2\sin^2\frac{x}{2}}}\, dx$$

$t = \sin\frac{x}{2}$

$dt = \frac{1}{2}\cos\frac{x}{2}\, dx$

$$\therefore I = \sqrt{2}\int_0^{\frac{1}{\sqrt{2}}} \frac{2dt}{\sqrt{1-2t^2}}$$

52. $I = \int_{-\pi/4}^{\pi/4} \log(\sec\theta - \tan\theta)\, d\theta$

$I = \int_{-\pi/4}^{\pi/4} \log(\sec\theta + \tan\theta)\, d\theta$

$2I = 0$

53. $I = \int_0^{\pi} \sqrt{\sin x} \,.\, \cos\frac{x}{2}\, dx$

$$I = \int_0^{\pi/2} \sqrt{\sin 2\theta}\,.\cos\theta\, d\theta \ldots \text{(i)} \qquad [\because \frac{x}{2} = \theta,\ dx = 2d\theta]$$

$$I = 2\int_0^{\pi/2} \sqrt{\sin 2\theta}\ .\sin\theta\, d\theta \ldots \text{(ii)}$$

$$\text{(i)} + \text{(ii)} \Rightarrow 2I = 2\int_0^{\pi/2} \sqrt{\sin 2\theta}\ (\cos\theta + \sin\theta)\, d\theta$$

take $\sin\theta - \cos\theta = t$

$(\cos\theta + \sin\theta)\, d\theta = dt$

54. $I = \int_0^1 \sqrt{x}\ \sqrt{1-\sqrt{x}}$

$\sqrt{x} = t$

$dx = 2tdt$

$$I = 2\int_0^1 t^2 \left(\sqrt{1-t}\right) dt$$

$$= 2\int_0^1 (1-t)^2 \sqrt{t}\ dt$$

55. $$I = \int_0^{\frac{\pi}{4}} \frac{\sin 2\theta}{1-\frac{1}{2}(\sin 2\theta)} d\theta$$

$\cos 2\theta = t$

$-2\sin 2\theta d\theta = dt$

$$I = \int_0^1 \frac{2dt}{1+t^2}$$

56. $$I = \int_0^{\pi} \frac{\sin 100x}{\sin x} dx$$

$$= \int_0^{\pi} \frac{\sin 100(\pi - x)}{\sin(\pi - x)} dx = -I$$

57. $I = \int_{-\pi/2}^{\pi/2} \sin x\, f(\cos x)\, dx$

$$= \int_{-\pi/2}^{\pi/2} \sin\left(-\frac{\pi}{2}+\frac{\pi}{2}-x\right) f\left(\cos\left(-\frac{\pi}{2}+\frac{\pi}{2}-x\right)\right) dx$$

$= -I$

58. $I = \int_{-1}^{1} (x^2 + x)\,|x|\,dx = \int_{-1}^{1} x^2\,|x|\,dx + \int_{-1}^{1} x\,|x|\,dx$

$= 2\int_{0}^{1} x^2 \cdot x\,dx + 0$ ($\because$ $x\,|x|$ is an odd function)

59. $I = \int_{0}^{\pi} [\cot x]\,dx = \int_{0}^{\pi} [\cot(\pi - x)]dx = \int_{0}^{\pi} [-\cot x]\,dx$

$2I = \int_{0}^{\pi} ([\cot x] + [-\cot x])\,dx$

$= \int_{0}^{\pi} (-1)\,dx$

[$\because$ $x \in R$ if x is an integer then $[x] + [-x] = 0$ and if x is not an integrer then $[x] + [-x] = -1$]

60. $f\left(\frac{1}{2}\right) - f\left(-\frac{1}{2}\right) = \int_{0}^{\frac{1}{2}} \log\left(\frac{1-t}{1+t}\right) dt - \int_{0}^{-\frac{1}{2}} \log\left(\frac{1-t}{1+t}\right) dt$

$$= \int_{0}^{\frac{1}{2}} \log\left(\frac{1-t}{1+t}\right) dt + \int_{-\frac{1}{2}}^{0} \log\left(\frac{1-t}{1+t}\right) dt$$

$$= \int_{\frac{-1}{2}}^{\frac{1}{2}} \log\left(\frac{1-t}{1+t}\right) dt$$

$= 0$ ($\because$ $\log\left(\frac{1-t}{1+t}\right)$ is an odd function t)

61. $cx = c^2 + t$

$$I = \int_{1+c}^{a+c} 1\, dx = c\,(a-1)$$

62. $7 = [(3-x) f'(x)]_2^3 - \int_2^3 (0-1) f'(x) dx$

$7 = 0 - f'(2) + f(3) - f(2)$

63. $\int_1^{\frac{1}{e}} \frac{\log t}{1+t}\, \mathrm{dt}$

$= -\int_1^e \frac{\log \frac{1}{u}}{u(u+1)} dy \qquad \left[\because t = \frac{1}{u} dt = -\frac{1}{u} dy\right]$

$= \int_1^e \frac{\log u}{u(u+1)}$

$\int_1^e \frac{\log t}{1+t} dt + \int_1^{\frac{1}{e}} \frac{\log t}{1+t} dt$

$= \int_1^e \frac{1}{t} \log t\, \mathrm{dt}$

64. $I = \int_0^\pi x\, f(\sin x)\, dx$

$= \int_0^\pi (\pi - x)\, f(\sin(\pi - x)\, dx$

$= \pi \int_0^\pi f(\sin x)\, dx - I$

65. $I = \int_{-\frac{3\pi}{2}}^{-\pi/2} [(x+\pi)^3 + \cos^2(x + 3\pi)]\, dx$

$= \int_{-\frac{3\pi}{2}}^{-\pi/2} \left[\left(-\frac{\pi}{2} - \frac{3\pi}{2} - x + \pi\right)^3 + \cos^2\left(-\frac{3\pi}{2} - \frac{\pi}{2} - x + 3\pi\right)\right] dx$

$$= -I + \int_{-\frac{3\pi}{2}}^{-\pi/2} (1 + \cos 2x)\, dx$$

66. $\int_{1/3}^{2/3} f\text{o}\, f(x)\, dx = \int_{1/3}^{2/3} \frac{1}{1-x}\, dx$

67. $\int_0^1 \frac{dx}{\sqrt{x}\,(x+1)}$ $\qquad \sqrt{x} = t$

$$= \int_0^1 \frac{2dt}{1+t^2} \qquad \frac{1}{2\sqrt{x}}\, dx = dt$$

68. $\int_0^{\frac{1}{2}} \frac{dx}{(1-x)^2 \sqrt{\frac{1+x}{1-x}}}$

$$= \int_0^{\sqrt{3}} dt \qquad \left[\frac{1+x}{1-x} = t^2,\ \frac{2}{(1-x)^2}\, dx = 2t\, dt\right]$$

69. $h(-x) = (f(-x) + g(-x))\,(g(-x) - f(-x))$

$= (-f(x) + g(x))\,(g(x) + f(x))$

$= h(x)$

$$\therefore \int_{-\pi/2}^{\pi/2} h(x)\, dx = 2\int_0^{\pi/2} h(x)\, dx$$

70. $\sum_{k=1}^{100} \int_0^1 f(x + k - 1)\, dx$

$$= \sum_{k=1}^{100} \int_{k-1}^{k} f(t)\, dt$$ [Putting x+k-1=t, dx=dt]

$$= \int_0^1 f(t)\, dt + \int_1^2 f(t)\, dt + \ldots.. + \int_{99}^{100} f(t)\, dt$$

$$= \int_0^{100} f(t)\, dt$$

71. $\int_{1/e}^{e} \frac{1}{x} f\left(x - \frac{1}{x}\right) dx$ $\qquad \because \frac{1}{x} = t \Rightarrow -\frac{1}{x^2}\,dx = dt$

$= \;= \int_{e}^{\frac{1}{e}} \frac{1}{t} f\left(\frac{1}{t} - t\right) dt$

$= \;= \int_{\frac{1}{e}}^{e} \frac{1}{t}\left[-f\left(t - \frac{1}{t}\right)\right] dt$

$= -\,\mathrm{I}$

72. Putting t = cosθ

$\int_{0}^{\pi/2} \sin\theta \log\left(1 - \cos^2\theta\right)^{\frac{1}{2}} d\theta = -\frac{1}{2}\int_{1}^{0} [\log(1 - t) + \log(1 + t)]\,dt$

73. $\int_{1}^{0} \log\left(\frac{1-x}{x}\right) dx$

$= \int_{0}^{1} \log(1 - x)\,dx - \int_{0}^{1} \log x\,dx$

$= \int_{0}^{1} \log(1 - (1 - x))\,dx - \int_{0}^{1} \log x\,dx$

$= 0$

74. $\mathrm{I} = \int_{0}^{\frac{\pi}{4}} \frac{2\sin x\cos x}{(\sin x + \cos x + 1)}\,dx$

$= \int_{0}^{\frac{\pi}{4}} \frac{2\sin x}{(\tan x + 1 + \sec x)}\left(\frac{\tan x + 1 - \sec x}{\tan x + 1 - \sec x}\right) dx$

$= \int_{0}^{\frac{\pi}{4}} (\sin x + \cos x - 1)\,dx$

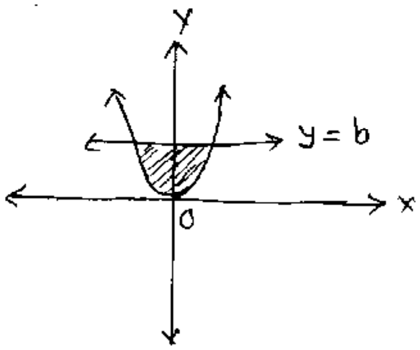
75. Take $x - \frac{1}{x} = t$

$\left(1 + \frac{1}{x^2}\right) dx = dt$

76. Applying integration by parts

77. $I = \int_0^b xdy$

$\frac{4}{3} = \int_0^b 2\sqrt{b}\,\sqrt{y}\,dy$



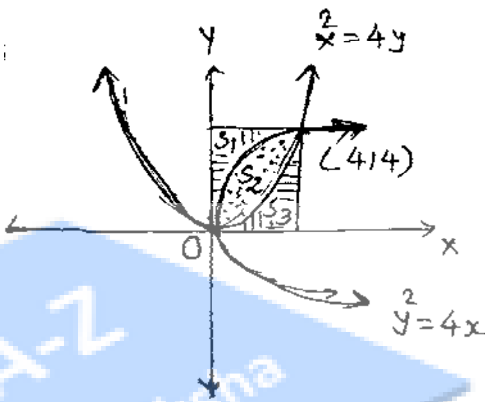
78. $S_1 = S_3$ ..... (i)

$\& S_2 = \int_0^4 \left(2\sqrt{x} - \frac{x^2}{4}\right) dx = \frac{16}{3}$

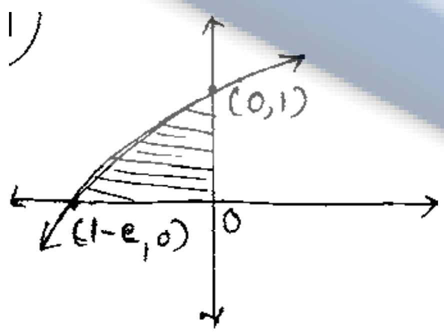
$S_1 + S_2 + S_3 = 4 \times 4$

$2S_1 = 16 - \frac{16}{3}$

$S_1 = \frac{16}{3}$ Sq. unit

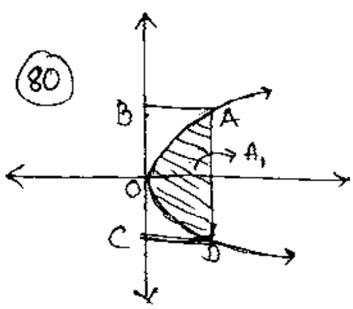


79. $A = \int_{1-e}^{0} \log_e (x + e)\, dx$



80. $A_1 = 2\,|\,I\,|$

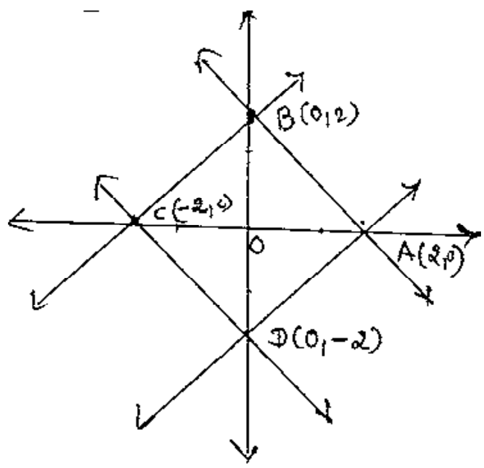
$I = \int_0^8 4\sqrt{2}\,\sqrt{x}\,dx$



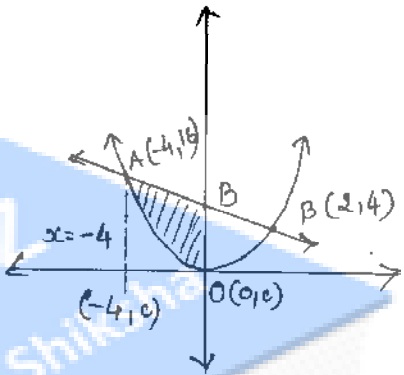
81. A = 4 | I |

$= 4 \int_0^2 (2 - x)\, dx$

OR $A = 4 \cdot \frac{1}{2} (2)(2)$



82. $I = \int_{-4}^{0} (8 - 2x - x^2)\, dx$



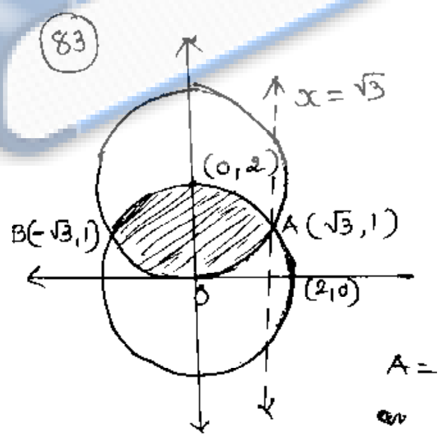
83. A = 2 | I |

$I \int_0^{\sqrt{3}} \left(\sqrt{2^2 - x^2} - 2 + \sqrt{2^2 - x^2}\right) dx$

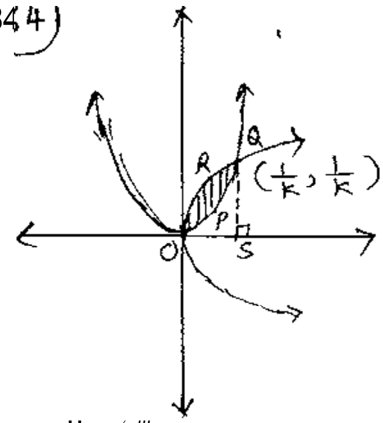
OR

A = 4 | I |

$I = \int_0^1 \sqrt{4 - (y-2)^2}\; dy$



84.



$$12\int_0^{\frac{1}{k}}\left[\sqrt{\frac{x}{k}}-kx^2\right]dx$$

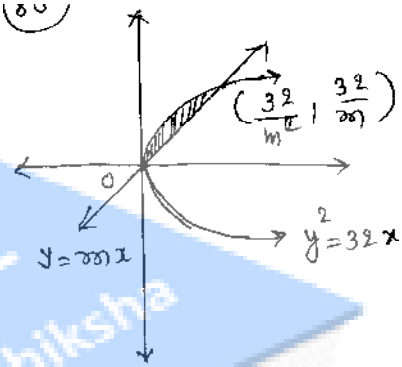
$$3k^2 = \frac{1}{12}$$

$$k = \frac{1}{6}\ (\because\ k > 0)$$

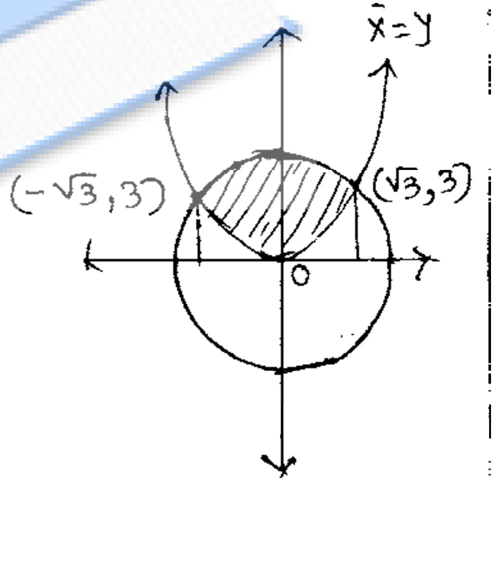
85. $\frac{8}{3} = \int_0 4\sqrt{2}\sqrt{x} - mx \quad dx$

$$\frac{8}{3} = \frac{512}{3m^3}$$

$$m = 4$$

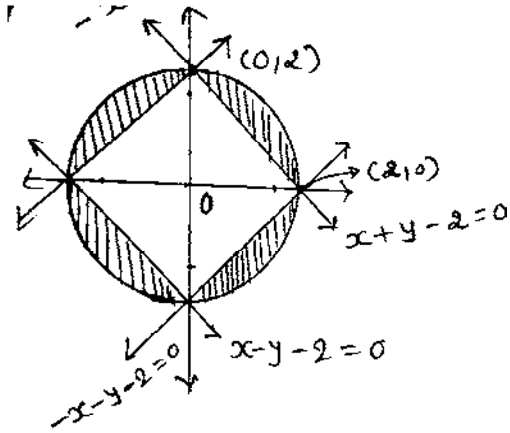


86. $A = 2\int_0^{\sqrt{3}} \sqrt{12 - x^2} - x^2 \quad dx$

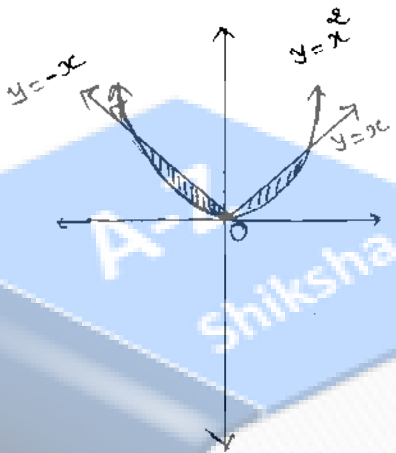


87. $A = \pi\,(2)^2 - \left(2\sqrt{2}\right)^2$

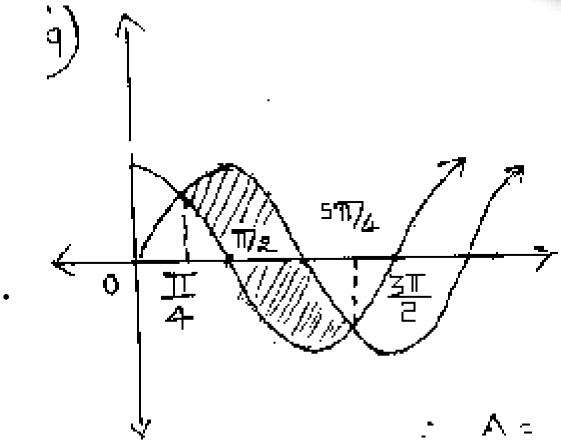
$= 4\pi - 8$



88. $A = 2\int_0^1 (x - x^2)\,dx$



89.



$\frac{\pi}{4} \le x \le \frac{5\pi}{4}$

$\Rightarrow \sin x \le \cos x$

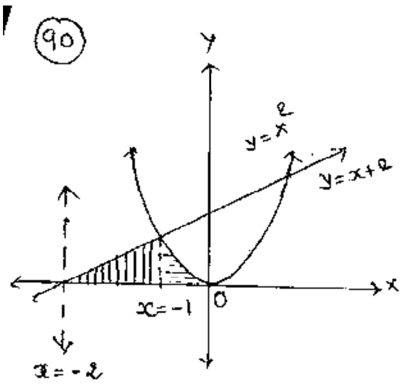
$\therefore\ A = \int_{\pi/4}^{5\pi/4} (\sin x - \cos x)\,dx$

$I_2 = \int_{-1}^{0} x^2 dx$

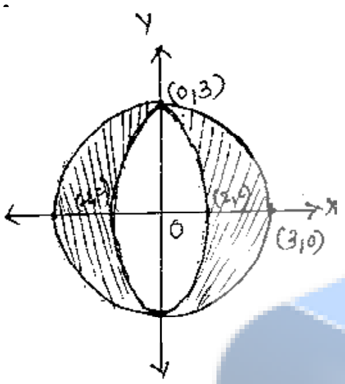
90. $A = |I_1| + |I_2|$

$$I_1 = \int_{-2}^{-1} (x+2)\,dx$$

$$I_2 = \int_{-1}^{0} x^2\,dx$$



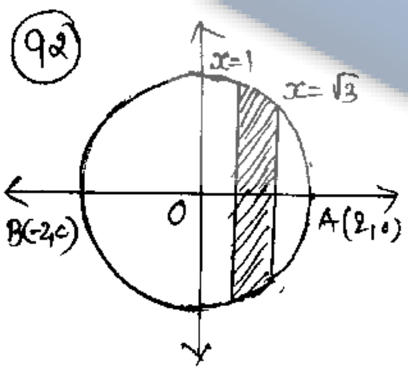
91.



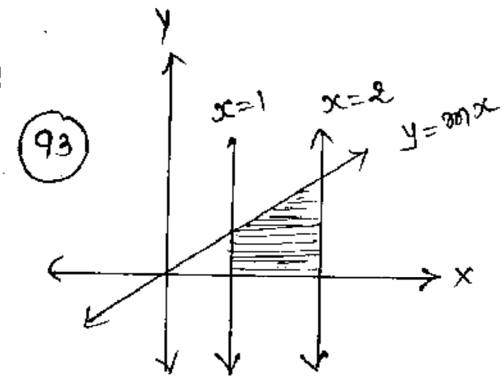
$$= 4\int_{0}^{3}\left(\sqrt{9-y^2} - \frac{2}{3}\sqrt{9-y^2}\right)dy$$

$$= \frac{4}{3}\int_{0}^{3} \sqrt{(3)^2 - y^2}\;dy$$

92.



$$A = 2\int_{1}^{\sqrt{3}} \sqrt{4 - x^2}\,dx$$



$$6 = \int_{1}^{2} mx\,dx$$

$$6 = m\left[\frac{x^2}{2}\right]_1^2$$

Answer

(1) b	(26) b	(51) d	(76) b
(2) b	(27) c	(52) d	(77) c
(3) b	(28) c	(53) c	(78) d
(4) d	(29) d	(54) d	(79) a
(5) b	(30) d	(55) c	(80) b
(6) d	(31) d	(56) a	(81) c
(7) b	(32) c	(57) c	(82) c
(8) b	(33) b	(58) b	(83) d
(9) c	(34) a	(59) c	(84) b
(10) c	(35) b	(60) a	(85) c
(11) b	(36) d	(61) b	(86) c
(12) d	(37) d	(62) d	(87) c
(13) d	(38) c	(63) d	(88) c
(14) b	(39) a	(64) d	(89) d
(15) b	(40) c	(65) b	(90) c
(16) c	(41) c	(66) d	(91) b
(17) d	(42) c	(67) c	(92) b
(18) a	(43) c	(68) c	(93) d
(19) d	(44) a	(69) d	
(20) c	(45) d	(70) b	
(21) a	(46) d	(71) d	
(22) b	(47) d	(72) a	
(23) a	(48) d	(73) c	
(24) c	(49) d	(74) b	
(25) c	(50) a	(75) a	